## Biostatistics

www.harding.edu/plummer/biostats/biostats.pdf
Spring 2018

| $\frac{\text { Introduction }}{\text { to Statistics }}$ | $\frac{\text { Descriptive }}{\text { Statistics }}$ | Graph <br> Construction | $\frac{\text { Inferential }}{\text { Statistics }}$ | $\frac{\text { Hypothesis }}{\text { Testing I }}$ | $\frac{\text { Hypothesis }}{\text { Testing II }}$ | $\frac{\text { Practice }}{\text { Problems }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Will I use }}{\text { this stuff? }}$ | $\frac{\text { Statistical }}{\text { Tables }}$ | $\frac{\text { Faculty }}{\text { Interests }}$ | $\frac{\text { Statistical }}{\text { Tests }}$ | $\frac{\text { Advanced }}{\text { Stat Tests }}$ | $\frac{\text { Protocol }}{\text { Sheet }}$ | Grades |

## Course Description

An introductory computer-based statistics course that includes instruction in SYSTAT. Topics covered include populations and samples, variables, probability distributions, descriptive statistics, statistical inference, and hypothesis testing. Included are selected parametric and non-parametric tests for examining differences in means, variances, and frequencies as well as correlation, regression, and tests of independence.

Emphasis is given to practical matters such as how to choose appropriate analyses and how to interpret results, both statistically and biologically. High school algebra is the only math background you need. Biostats is a practical application course - to learn it, you have to do it. Failing to apply statistical concepts and procedures on a regular basis will diminish your chances of understanding the material and earning the grade you desire.

## What we have to learn to do, we have to learn by doing. - Aristotle

Student Learning Outcomes - By the end of the semester you will be able to:

- understand how science and statistics interact
- apply basic statistical procedures using professional statistical software
- read and understand primary biological literature


## Textbooks and Software

- Primary text - www.harding.edu/plummer/biostats/biostats.pdf
- Supplementary text - www.khanacademy.org/math/statistics-probability
- Primary software - SYSTAT (provided on computers in S161 and S182).
- Student software - MYSTAT (free student version of SYSTAT; download at www.systat.com)


## Evaluation

| Exam 1 | 20\% | Exams 1-3 are comprehensive and consist of Content (scantron/short answer $\sim 50 \%$ ) and Practical (SYSTAT problems/graphing $\sim 50 \%$ ) sections. An extra point may be earned on each exam if you are present in class when feedback is given on your graded exams. Exam study guides |
| :---: | :---: | :---: |
| Exam 2 | 20\% |  |
| Exam 3 | 20\% |  |
| Quizzes | 20\% | $\sim 10$ announced quizzes and exercise |
| Final <br> Exam | 20\% | The final exam is a comprehensive scantron exam taken during the regularly scheduled final exam period. Unlike Exams 1-3, you will not use a computer on the final for any task; this includes SYSTAT. Exam study guides |

## Classroom Policies

- Computer resources that may be viewed during lecture include the course website, SYSTAT, and your M-drive. All other uses (e.g., social notworking sites such as Facebook, Twitter, Instagram, email, blogs, sports news, pictures of your girl/boyfriend, etc.) are off limits during lecture.
- Cell phone use during lecture is prohibited. If you must send or receive a text or call during lecture time, please excuse yourself from the classroom and take it to the hallway.
- Regular class attendance is necessary to do well in this course. Excessive unexcused absences will be handled on an individual basis. An official HU class excuse or prior arrangements with the instructor is necessary to be excused from an exam.
- Cheating in all its forms is inconsistent with Christian faith and practice and will result in sanctions up to and including dismissal from the class with a failing grade. Instances of dishonesty will be handled according to the procedures delineated in the Harding University catalog.
- The visual appearance or use of any unapproved electronic device during an exam will be interpreted as cheating and will result in a zero for that exam.
- In accordance with the official Time Management Policy of the University, you are expected to spend two hours outside of class for each credit hour spent in class each week. That amounts to six additional hours per week, two of which are imposed on you in conjunction with regular class time.
- THE ONLINE BIOSTATS LECTURE NOTES ARE NOT COMPLETE SOURCES OF INFORMATION FOR EXAMS. In general, students are responsible for anything discussed in class.


## My Responsibilities

Because, as your teacher, I have a substantial responsibility to you and to the Lord (James 3:1), I promise my best effort to you in Biol. 254. I pray that my lectures will be clear, my expectations reasonable, and my exams vigorous, thorough, challenging, and fair. I also pray that your grade will reflect both your ability and your preparation. Finally, I hope that you will learn something substantive in my class regardless of what you think about the subject matter. For further insight into my teaching philosophy, click here - Good luck!

## Misc.

- You will need a personal Dropbox account. Data files for the course are available in a shared Dropbox folder called "Student Biostats." You should download these files to your M-drive.
- Statements on academic dishonesty, teaching evolution, and students with disabilities


## WILL I EVER USE THIS STUFF?

You may be thinking (and perhaps hoping?) you'll never have to use statistics. The reality is that if you become a professional of any kind, you will very likely use statistics according to at least one of the three objectives of this course. Do yourself a favor and read the unsolicited testimonials from former biostats students.

## Introduction to Statistics

Home

## The relationship of science and statistics

A. Process of science represents an interplay of ideas and data (ID); BC ideas only

1. Use data to make inferences (decisions) relative to ideas [=analyze data]
2. Statistics is a tool that assists decision-making
3. Great increase in use of statistics

- basic science: American Naturalist
- applied science: "Evidence-based medicine (EBM) is an approach to medical practice intended to optimize decision-making by emphasizing the use of evidence from well-designed and carefully-conducted research" (Wikipedia); Textbook

4. Manufacturing-based society $\rightarrow$ service/information-based society

- great need for quantitative methods of making decisions using information available
- e.g., demo quantitative decision-making with scatterplot
B. Levels of organization within biology and the relative use of statistics

| Community biology $\uparrow$ | -Greater need for stats |
| :--- | :--- |
| Population biology | -More uncertainty |
| Organism biology | -Less understanding |
| Organ biology | -More variation |
| Cell biology | -More factors (=more |
| Molecular biology | complexity) |

How do we know what we know? - mechanics of the process of science
5. R.B. Fischer - "Science is what scientists do when they're working."
6. What DO scientists do when they are working?
7. Several processes: HYPOTHETICO-DEDUCTIVE (IF-THEN) MODEL

```
OBSERVATION }->\mathrm{ HYPOTHESIS }->\mathrm{ PREDICTION }->\mathrm{ DESIGN TEST }
                                    IF }->\mathrm{ THEN -observation
                                    -experiment
COLLECT DATA }->\underset{\mathrm{ (statistics) }}{\mathrm{ ANALYZE DATA }}->\underset{\mathrm{ (probability) }}{\mathrm{ CONCLUSIONN}}->\mathrm{ (talks, publications)
-> REFINE AND GENERALIZE HYPOTHESIS }->\mathrm{ THEORY (= model)
Absolute certainty is a privilege of uneducated minds-and fanatics. It is, for scientific folk, an unattainable ideal. - C. J. Keyser
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8. Advancement comes by disproving false hypos; "proof" in science means disproving false hypotheses

- A. Einstein - "No amount of experimentation can ever prove me right; a single experiment can at any time prove me wrong."
- D. Hull - "The scientific method does not guarantee that you are right; it guarantees that if you are wrong, someone will find it out."
- S. Connery - "Isn't that what science is all about,...eliminating possibilities?" (video)

9. "proof" is tentative - most models have historically been either discarded or radically modified - no reason to believe that it will be different in the future

- scientific models are pragmatic (useful) - if model works, use it!
- scientific "truth" - not necessarily "TRUTH"

What constitutes the study of "statistics?" (often misunderstood)
-e.g., "There are three kinds of lies: lies, damn lies, and statistics." -B. Disraeli


## What Statistics Is Really About

| Population <br> numerical properties <br> "parameters" |  | Sample <br> numerical properties $=$ <br> "statistics" |
| :--- | :--- | :--- |
| The population is what we want to <br> understand. | "Error" | The sample is what we use to understand the <br> population. <br> - Descriptive stats: describe data in the sample <br> - Inferential stats: infer from sample to |

## Descriptive Statistics <br> \section*{Home}

## Statistical Basics

A. Definitions

1. variable - characteristics that may differ (vary) among individuals
a. measured
b. derived (non-measured); derived from measured variables
c. dependent/response variable vs. independent/predictor variable
2. data - values of variables for individuals (singular datum)
3. case/observation - an individual; symbolize: $\mathrm{x}_{1}, \mathrm{x}_{2}, \ldots \mathrm{x}_{\mathrm{n}}$ ( $\mathrm{n}=$ sample size)
B. Collection of data
4. population - all individuals of a defined universe (= whatever we say it is!)
5. sample-subset of population; used to make inferences regarding the population
6. statistical error-difference between the real population value and the estimates (from sample data) of the population value
7. randomness - all individuals have equal probability of being sampled
8. independence - value of one case does not affect the value of other cases
C. Scales of measurement and variable types
9. Categorical scale (Nominal)
a. values not quantitative or ranked; no mathematical or value relationship
b. mutually exclusive categories (e.g., male/female)
c. 1 variable type: categorical
10. Ranked scale (Ordinal)
a. relative differences (e.g., greater than/less than)
b. no mathematical relationship between values (e.g., small/medium/large; highly active/active/not active)
c. 1 variable type: ranked
11. Ratio scale
a. mathematically defined distance between values; quantitative
b. absolute zero point (e.g., mass)
c. 2 variable types:

- Discrete - may assume only certain values within given range (e.g., 1, 2, 3, 4)
- Continuous - may assume any value within given range (e.g., 1.0, 2.34, 2.344)
d. may convert ratio data to ranked/categorical data (but not vice versa)

4. Interval scale
a. mathematically defined distance between values; quantitative
b. arbitrary zero point (e.g., Celsius temperature scale)
c. 2 variable types:

- Discrete - may assume only certain values within given range (e.g., 1, 2, 3, 4)
- Continuous - may assume any value within given range (e.g., 1.0, 2.0, 2.34, 2.344, etc.)
d. may convert interval data to ranked/categorical data (but not vice versa)
D. Identify variables and measurement scale (variable ID practice)


## E. SYSTAT Demo

1. windows (output, data, graph); menus
2. data files

- columns (variables); numerical vs. string (categorical) variables (e.g., SEX vs. SEX\$)
- rows (values of variables [cases, observations, sample size])

3. creating data files (entering and editing data)

- raw data file (stacked [=indexed] vs. unstacked data

4. opening existing data files (.SYZ files)
5. graphing frequency distributions (Graph $\rightarrow$ Histogram)
6. creating frequency tables (Analyze $\rightarrow$ One-Way Frequency Tables;
(Analyze $\rightarrow$ Tables $\rightarrow$ Two-Way)
7. calculating an average (Analyze $\rightarrow$ Basic Statistics)
8. selecting cases (Data $\rightarrow$ Select Cases)
9. analyze by groups (Data $\rightarrow$ By Groups); groups $=$ categories
10. transforming data (Data $\rightarrow$ Transform $\rightarrow$ Let and Data $\rightarrow$ Transform $\rightarrow$ If..,Then Let

## Introductionto SYSTAT

Prepare a SYSTAT data file using the data below. These data are measurements taken from 10 specimens of spiny guanotzits from Arkansas and Missouri. The variables are: collection locality (categorical), length of body (continuous), sex (categorical), weight of body (continuous), amount of pigment on the lower jaw (ranked), and number of scales on the chin (discrete).

| Case | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Locality | AR | AR | MO | MO | MO | AR | AR | MO | AR | MO |
| Length $(\mathrm{mm})$ | 22.5 | 21.4 | 20.8 | 20.6 | 19.8 | 20.1 | 22.3 | 21.7 | 20.4 | 21.1 |
| Sex | m | m | f | f | f | f | m | f | m | f |
| Weight $(\mathrm{g})$ | 333 | 298 | 401 | 257 | 21 | 30 | 478 | 400 | 35 | 288 |
| Pigment | 4 | 5 | 5 | 3 | 2 | 1 | 1 | 5 | 4 | 5 |
| No. scales | 23 | 22 | 14 | 26 | 9 | 21 | 17 | 12 | 15 | 12 |

Name your data file first.syz (the file extension .syz identifies a SYSTAT data file). After you finish entering the data, proofread the file to make sure that the data are correct, edit if necessary, save the file and close it. Reopen the file and use it to learn the following menus and functions:

- File Menu (New, Open, Save, Save As, Print, Exit)
- Edit Menu (Undo, Cut, Copy, Paste, Copy Graph, Delete, Options)
- Data Menu (Variable properties, Transform [Let and If - Then Let], By Groups, Select Cases)
- Graph Menu (Histogram)
- Analyze Menu (One-Way Frequency Tables, Basic Statistics, Tables)


## Exercises

1. calculate the average guanotzit weight ( 254.1 g )
2. calculate the average guanotzit weight separately for males and females ( $\mathrm{m}=286.0 \mathrm{~g} ; \mathrm{f}=232.8 \mathrm{~g}$ )
3. calculate the average weight for guanotzits from Arkansas (234.8g)
4. draw a histogram of guanotzit lengths
5. transform weight to the common logarithm of weight

(case 1: 333.0 to 2.522)
6. create the new variable USE\$ and let its value ("yes" or "no") be determined by a combination of values of the variables SEX\$ and LOC\$. Example: If SEX\$="m" and LOC\$="AR"... Then Let USE $=$ "yes." Notice that the variable USE\$ is a derived variable, not a measured variable
7. how many quanotzits from Missouri were measured? ( $\mathrm{n}=5$ )
LOC\$ by NOSCALES

| $\mathbf{9}$ | $\mathbf{1 2}$ | $\mathbf{1 4}$ | $\mathbf{1 5}$ | $\mathbf{1 7}$ | $\mathbf{2 1}$ | $\mathbf{2 2}$ | $\mathbf{2 3}$ | $\mathbf{2 6}$ | Total |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| AR | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 5 |
| M | 1 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 5 |

8. determine the number of guanotzits by scale number and state

## ********************************************

## Description of Data (from a frequency distribution)

A. Descriptive statistics

1. measures of central tendency
a. mode - most frequent class (of frequency distribution)
b. median (ordinal or ratio/interval data) - middle class
c. mean (ratio/interval data) $=$ "average"; $\Sigma x / n$
d. weighted mean (ratio/interval data) - $\Sigma \mathrm{fx} / \mathrm{n}$; used when cases have different levels of importance (weights); e.g., grade point average
2. measures of dispersion - describe the amount that each observation is likely to vary from the mean/median
a. maximum, minimum (range): sensitive to extreme values
b. interquartile range: (quartiles, middle $50 \%$ of observations (Q3 - Q1; difference between $25^{\text {th }}$ and $75^{\text {th }}$ percentiles)
c. sum of squares(SS): $\Sigma(\mathrm{x}-\overline{\mathrm{x}})^{2}$

d. variance: $S S / n$
e. standard deviation: $\sqrt{ }$ variance
3. symbols for statistics (sample) and parameters (population)

|  | Parameter | Statistic |
| :--- | :--- | :--- |
| Mean | $\mu=\sum \mathrm{x} / \mathrm{n}$ | $\overline{\mathrm{x}}=\sum \mathrm{x} / \mathrm{n}\left(={ }^{\prime \prime} \mathrm{x}-\mathrm{bar"}\right)$ |
| Variance | $\sigma^{2}=\Sigma(\mathrm{x}-\mu)^{2} / \mathrm{n}$ | $\mathrm{s}^{2}=\sum(\mathrm{x}-\overline{\bar{x}})^{2} / \mathrm{n}-1$ |
| Standard Deviation | $\sigma=\sqrt{ } \sigma^{2}$ | $\mathrm{~s}=\sqrt{ }\left(\mathrm{s}^{2}\right)(="$ SD" $)$ |

4. coefficient of variation (CV)
-expresses SD as a percent of the mean a. $\mathbf{C V}=(\mathbf{S D} / \overline{\mathbf{x}}) \mathbf{1 0 0}$
-used to compare relative variation in one variable between groups with different means
Example:

|  | mean | SD | CV | Note that group 2 is relatively |
| :--- | :--- | :--- | :--- | :--- |
| more variable despite a greater |  |  |  |  |
| Group 1 | 14.2 | 2.5 | 17.6 | mon |
| Group 2 | 7.2 | 1.8 | 25.0 | SD in group 1. |

B. Calculating Descriptive Statistics (mean $\pm$ SD)

1. calculate descriptive statistics from raw data file; Analyze $\rightarrow$ Basic Statistics
-Use CAVESALYS.SYZ (Sanders)
QUESTION: What are the descriptive statistics of snout-vent-length for female salamanders collected in Arkansas?
2. calculate descriptive statistics from frequency distribution

- Step 1: Data $\rightarrow$ Case Weighting $\rightarrow$ By Frequency
- Step 2: Analyze $\rightarrow$ Basic Statistics
$\left.\begin{array}{|l|l|}\hline \begin{array}{l}\text { Value of } \\ \text { Variable }\end{array} & \begin{array}{l}\text { No. times } \\ \text { observed }\end{array} \\ \hline 0 & 7 \\ \hline 1 & 24 \\ \hline 2 & 93 \\ \hline 3 & 99 \\ \hline 4 & 24 \\ \hline & \text { Total }\end{array}\right) 247$.
C. How to report sample means (must include a measure of error)
a. Text (example)
b. Tables (example)
c. Graphs (error bars; example)
-how reduce variance? $(\Sigma(\mathbf{x}-\overline{\mathbf{x}}) / \mathbf{n})$
-what limits n? (availability, money, time)

You now have sufficient knowledge to begin the Graph Construction Exercise on p15.
$\underline{\text { Probability Distributions (expected probabilities associated with all possible outcomes) }}$

- cannot know if experimental result is due to chance alone unless we know what the expected is (hypothesis testing - basis for much of much of this course!)
- basic question: How well does an observed frequency distribution fit an expected frequency distribution? (goodness of fit - GOF)

Discrete probability distribution - Binomial (mutually exclusive categories; either/or); e.g., male/female, red/white, red/not red

## Probability Basics

Example: 1 coin toss- possibilities: $1 \mathrm{H}, 1 \mathrm{~T}$
a. probabilities: no. ways an event (H or T) can occur /total no events (2) possible; "division" rule; $1 \mathrm{H}[1 / 2]=0.5 ; 1 \mathrm{~T}[1 / 2]=0.5$
b. add all possibilities $=1[0.5+0.5=1]$
c. probability distribution shape


Example: 2 coin toss- possibilities: $2 \mathrm{H}, 1 \mathrm{H} 1 \mathrm{~T}, 1 \mathrm{~T} 1 \mathrm{H}, 2 \mathrm{~T}$ (mutually exclusive, independent events)
a. probabilities:

1) simultaneous events ("and" rule, multiply): 2 H [ $0.5 \mathbf{x}$ $0.5]=0.25 ; 2 \mathrm{~T}[0.5 \times 0.5]=0.25$
2) alternative events ("or" rule, add): $2 \mathrm{HT}[0.5 \times 0.5]+$ $[0.5 \times 0.5]=0.5$
b. add all probabilities $[0.25+0.5+0.25=1]$
c. probability distribution shape


## Binomial Distribution

1. formula: $\mathbf{P}(\mathbf{x})=(\mathbf{n}!/(\mathbf{x}!(n-x)!)) \mathbf{p}^{\mathbf{x}_{\mathbf{q}}(\mathbf{n}-\mathbf{x})}$
-no need to memorize the formula but you must be able to recognize the formula and each of its terms
2. terms
$\mathrm{P}=$ probability of the number of occurrences of the event of interest
$\mathrm{p}=$ probability of event of interest $=$ head ("success")
$\mathrm{q}=$ probability of other event $(1-\mathrm{p})=$ not head ("failure")
$\mathrm{n}=$ number of "simultaneous" events (trials)
$x=$ number of occurrences of the event of interest
3. binomial shape determined by values of $n$ and $p$


EXAMPLE: A reproductive physiologist counted the number of males in 247 litters of 4 siblings each in a species of Dimetrodon (Table). Do these data support the hypothesis of sex being determined by a XX, XY system as occurs in mammals?

| No. males <br> Observed | Observed <br> frequency | Expected <br> frequency |
| :--- | :--- | :--- |
| 0 | 7 |  |
| 1 | 24 |  |
| 2 | 93 |  |
| 3 | 99 |  |
| 4 | 24 |  |
| Total | $\mathbf{2 4 7}$ | $\mathbf{2 4 7}$ |



Based on the theory of sex determination in mammals (equal chance of being male or female), calculate the expected frequencies for the number of males in these litters.

| $\underline{P(x)}=\left(n!/(x!(n-x)!) p^{x} q^{(n-x)}\right.$ | Expected proportion | No. litters | Expected number(frequency) |
| :---: | :---: | :---: | :---: |
| $\operatorname{prop}(0$ males $)=(4!/(0!(4-0)!)) \times 0.5{ }^{0} \times 0.5^{(4-0)}=$ | 0.0625 | $247=$ | 15.438 |
| prop $(1 \mathrm{male})=(4!/(1!(4-1)!)) \times 0.5{ }^{1} \times 0.5^{(4-1)}=$ | 0.2500 | $247=$ | 61.750 |
| prop $(2$ males $)=(4!/(2!(4-2)!)) \times 0.5^{2} \times 0.5^{(4-2)}=$ | 0.3750 | x $247=$ | 92.625 |
| prop $(3$ males $)=(4!/(3!(4-3)!)) \times 0.5^{3} \times 0.5^{(4-3)}=$ | 0.2500 | x $247=$ | 61.750 |
| prop $(4$ males $)=(4!/(4!(4-4)!)) \times 0.5{ }^{4} \times 0.5^{(4-4)}=$ | $\underline{0.0625}$ | $247=$ | 15.438 |
| Total | 1.00 |  | 247 |

## $\underline{\text { SYSTAT calculation of expected frequencies (Utilities } \rightarrow \text { Probability Calculator } \rightarrow \text { Univariate }}$

 Discrete)Question: Is the sex of Dimetredon determined by a mechanism similar to that of mammals? Expect 1:1. Compare observed with expected.

| No. males <br> Observed | Observed <br> frequency | Expected <br> frequency | Conclusion: because of <br> the large deviations |
| :--- | :--- | :--- | :--- |
| 0 | 7 | 15.438 | between the expected |
| bend observed numbers, |  |  |  |

So, what determines sex in Dimetredon?

Importance of sample size for observed data ( 1 coin example, compare to theoretical)

- IF observed = norm coin, THEN the larger the n , the closer we approximate expected conversely, THEN the smaller the n , the more we deviate from expected


## Exercise:BinomialDistribution

Assuming that the sex of hatchling turtles is determined by a particular combination of chromosomes as in mammals (i.e., an XX, XY system), fill in the expected frequencies below:

Data are number of male hatchlings emerging from 84 nests of kaw turtles (kaw turtles always lay 6 eggs per nest).
$\left.\begin{array}{|l|l|l|l|}\hline \begin{array}{l}\text { No. Males } \\ \text { Observed }\end{array} & \begin{array}{l}\text { Observed } \\ \text { No. Nests }\end{array} & \begin{array}{l}\text { Expected } \\ \text { No. Nests }\end{array} & \begin{array}{l}\text { Compare the observed and } \\ \text { expected frequencies. Do these } \\ \text { data support the hypothesis that }\end{array} \\ \hline 0 & 4 & & \\ \hline 1 & 7 & & \\ \text { sex of hatchlings is genetically } \\ \text { determined? (yes or no) }\end{array}\right\}$


Discrete probability distribution - Poisson (expected distribution for rare and random events)

1. Poisson: $\mu=\sigma^{2}\left(\sigma^{2} / \mu=1\right)$ - distribution defined by mean only; low value (rare events; e.g., recapture rates, bacterial viruses infecting bacteria)
2. Poisson formula: $\mathbf{P}(\mathbf{x})=\left(\overline{\mathbf{x}} \mathbf{x}^{-\overline{\mathbf{x}}}\right) / \mathbf{x}$ !
-Students: no need to memorize the formula but you must be able to recognize the formula and each of its terms
3. terms
$-\mathrm{P}=$ probability of the number of occurrences of the event of interest

- $\bar{x}=$ mean occurrence of event of interest
- e = mathematical constant (=2.71828)
- $\mathrm{x}=$ number of occurrences of the event of interest


6. Poisson shape determined by $\overline{\mathrm{x}}$

Example: An ecologist counted the number of maple seedlings in 100 quadrats

| No. <br> Plants | Obs. No. <br> Quadrats | Exp. No. <br> Quadrats |
| :--- | :--- | :--- |
| 0 | 35 |  |
| 1 | 28 |  |
| 2 | 15 |  |
| 3 | 10 |  |
| 4 | 7 |  |
| 5 | 5 |  |
| Total | $\mathbf{1 0 0}$ | $\mathbf{1 0 0}$ |



Using the mean calculated from the observed frequency distribution of maple seedlings per quadrat in the table $(\mathrm{x}=1.41)$, calculate the expected frequencies assuming that occurring in a quadrat is a random event.

Expected Expected proportion number (frequency)
prop $(0$ seedlings $)=\left(1.41^{0} \mathrm{e}^{-1.41}\right) / 0!=\quad \mathbf{0 . 2 4 4} \times 100=\mathbf{2 4 . 4 1}$
$\operatorname{prop}(1$ seedling $)=\left(1.41^{1} \mathrm{e}^{-1.41}\right) / 1!=\quad \mathbf{0 . 3 4 4} \times 100=\mathbf{3 4 . 4 2}$
-etc.
SYSTAT calculation of expected frequencies (Utilities $\rightarrow$ Probability Calculator $\rightarrow$ Univariate Discrete)

Question: Do seedlings occur randomly in quadrats?

| No. <br> Plants | Obs. No. quadrats | Exp. No. Quadrats | Conclusions: <br> 1. Is it rare? (mean=1.41) <br> 2. Is it random? <br> a. compare obs and exp distributions <br> b. calculate variance/mean ratio ( $2.18 / 1.41=1.55$ ) |
| :---: | :---: | :---: | :---: |
| 0 | 35 | 24.41 |  |
| 1 | 28 | 34.42 |  |
| 2 | 15 | 24.27 |  |
| 3 | 10 | 11.41 |  |
| 4 | 7 | 4.02 |  |
| 5 | 5 | 1.11 |  |
| Total | 100 | 100 |  |

## Exercise:PoissonDistribution

Assuming that being killed by a horse is a rare and random event, fill in the expected frequencies below.

Men killed by being kicked by a horse in the Prussian Army Corps.


Compare the observed and expected frequencies.
Do these data support the hypothesis that the chance of being killed by a horse in the Prussian Army Corps is a rare and random event? Support your conclusion.

## Exercise: Testing Your Concept of Randomness

1. draw 100 dots on the $10 \times 10$ grid on the next page (keep your eyes open, try to place dots randomly
2. count the number of cells with different numbers of dots
3. create a frequency table of your data
4. calculate the mean and variance of the number of dots per cell
```
mean = variance =
```

5. calculate the variance/mean ratio $=$
6. interpret: ratio $=1$ (random); ratio $<1$ (evenly spaced); ratio $>1$ (clumped)
7. Application: patterns of distribution in space reflect biological processes; for example, disease spread and behavioral/ecological interactions

## Patterns of Distribution




## Graph Construction <br> Home

In this exercise, you will learn to construct five basic graphs used by biologists. The rules for graph construction presented here will apply to all graphs you construct during the semester. As you finish graphs, copy and paste each image to a Word file named graphexercise, add the caption, and save. There are three parts to the exercise:

1. You will reproduce 5 finished graphs given to you;
2. You will be given data and asked to construct 5 appropriate graphs;
3. You will find an example of each of the 5 graph types in the primary literature.

## A. Basic graph types

1. Histogram (Graph $\rightarrow$ Histogram) - plots the frequency (counts/proportions/percentages) of occurrence as a bar on the Y -axis against a variable on the X -axis
2. Bar (Graph $\rightarrow$ Bar $)$ - plots the mean and error bars of a variable as a bar on the Y-axis against a categorical variable on the X -axis
3. Dot (Graph $\rightarrow$ Summary Charts $\rightarrow$ Dot $)$ - plots the mean and error bars of a variable as a symbol on the Y -axis against a categorical variable on the X -axis
4. Box Plot (Graph $\rightarrow$ Box Plot) - plots the median and quartiles of a variable on the Y -axis against a categorical variable on the X -axis
5. Scatterplot (Graph $\rightarrow$ Scatterplot) - plots cases of one variable on the Y-axis against cases of another variable on the X -axis

## Requirements of all graphs

- The Y variable is always read before the X variable. For example, "plot Y against X ", "plot Y by X ", and " Y is regressed against X ". For this class, X is never plotted against Y .
- Essential graph elements: axes (Y, X), axis labels (with units of measurement, if applicable), ticks, tick labels, caption
- Elements essential for specific graph types: bars, symbols, error bars, data points, line, linear smoother
- Each graph must be self-explanatory and be able to stand alone (figure captions are considered part of the graph). Captions should be descriptive, not interpretative.
- Non-standard abbreviations must be defined.
- Graphs displaying means (Bar, Dot) must portray the mean, error bars, and sample size for each mean.
B. Graph reproduction - Reproduce each graph (1-5) illustrated below. Read the description of each data file before beginning. Copy and paste your SYSTAT output into a Word file named graphexercise, add captions, and save.

1. HISTOGRAM - A SYSTAT Histogram plots the frequency (counts/proportions/percentages) of a single variable. Duplicate the Histogram below. Note axis titles, axis ranges, data plotted, bar fill, etc. The data are in RANDOM.SYZ (Plummer).


Fig. 1. The distribution of captures of green snakes according to location.
2. BAR - A SYSTAT Bar graph plots the mean of one variable against another variable.

Duplicate the BAR graph below. Note bar fill, axis titles, error bars, data plotted, etc. The data are in MOUSEDIET.SYZ (Cooper).


Fig. 2. The relationship of mean body mass and diet in laboratory mice fed different diets. Plotted are mean $\pm 1$ SD. Sample sizes are: $5 \mathrm{~K}-96$, n=34; AINcas, $n=35$; AIN-spi, n=32; P5001, n=42.
3. DOT - A SYSTAT Dot graph plots the mean of one variable against a discrete or categorical variable. Duplicate the Dot graph below. Note symbols, error bars, fill, axis titles, axis ranges, data plotted, etc. The data are in WORMSURVIVE.SYZ (JMGoy).


Fig. 3. Mean number of C. elegans exhibiting unimpaired movement according to trial day. Plotted are mean $\pm 1$ SD. Sample sizes are day $1, \mathrm{n}=48$; day $2, \mathrm{n}=51$; day 3, $\mathrm{n}=49$; day $4, \mathrm{n}=15$; day $5, \mathrm{n}=7$; day $6, \mathrm{n}=2$.
4. BOX - A SYSTAT Box Plot plots the quartiles of one variable against a discrete or categorical variable. Duplicate the Box Plot below. Note symbols, axis titles, axis ranges, selected data plotted, etc. The data are in CAVESALYS.SYZ (Sanders).


Fig. 4. Box plot of the body lengths of female Eurycea lucifuga captured in Arkansas and Kentucky caves in February and March. Plotted are the median (horizontal line), the $25^{\text {th }}$ and $75^{\text {th }}$ quartiles (box) and the maximum and minimum values (whiskers).
5. SCATTERPLOT - A SYSTAT Scatterplot plots individual cases of one variable against another variable. Duplicate the scatterplot below. Note symbols, axis titles, axis ranges, selected data plotted, etc. The data are in LONOKE.SYZ (Plummer).


Fig. 5. The relationship of body weight and snout-vent length in 99 adult (=individuals $>50 \mathrm{~cm}$ SVL) male diamondback water snakes.
C. Graph construction: Construct an appropriate graph for each of the following problems and save in your graphexercise file.
6. Use the following data on bill lengths ( mm ) of 42 belted kingfishers to construct a graph (Fig. 6) that plots the median and other quartiles separately for males, females, and the sexes combined (3 groups).

$$
\begin{gathered}
\text { males: } 48.1,47.7,48.0,50.6,50.8,49.9,49.3,50.8,46.9,49.9,48.8, \\
\text { 47.5, } 48.2,51.0,48.8,52.0,51.8,51.0,50.1,47.7,49.9 \\
\text { females: 53.8, 59.2, 52.3, 59.3, 56.5, 56.2, 55.6, 57.7, 52.5, 47.8, } \\
51.5,55.8,57.5,56.8,47.0,50.4,58.0,61.2,56.5,59.3,59.2
\end{gathered}
$$

For graphs 7-10, use the data file LONOKE.SYZ (Plummer).
7. Construct a graph (Fig. 7) that plots cases of weight against length for snakes collected in ponds \#53 and \#54. Indicate sample size.
8. Construct a graph (Fig. 8) that illustrates the mean body weight for each sex. Restrict cases to snakes $\geq 30$ and $\leq 90 \mathrm{~cm}$ SVL. You can more easily make the X-axis readable by creating a derived variable with this transform: IF sex=1 THEN LET sex $\$=$ "male"
9. Construct a graph (Fig. 9) that illustrates the frequency of female snakes captured in minnow ponds by snout-vent length. Indicate sample size.
10. Transform variable WGT with common logarithms. Construct a graph (Fig. 10) that plots cases of the transformed variable against SVL. Indicate sample size.
D. Literature Graphs: The third part of this exercise consists of finding an example of each of the five graph types in primary literature papers.

What is the Primary Literature? - Journals (evidence-based science; ID)

1. Original research written by the researcher
2. Peer reviewed
3. Publishing process
4. Some useful working categories
a. First tier-Science, Nature

- Broad subject content
- Publish only the best of the best
- Papers usually report a major advance in the field
b. Second tier—Proceedings of the National Academy of Sciences, Ecology, Cell
- Content frequently has restricted subject areas

- Publish most of the top papers in that subject area
- Reject many technically sound papers if they do not advance our knowledge sufficiently
c. Third tier—Journal of Herpetology, American Midland Naturalist, Journal of Immunology
- Content limited in subject area and/or geographical coverage
- Publish the bulk of papers in the subject area
- Most technically sound papers are accepted even if they do not dramatically advance our knowledge


## Structure of a Primary Literature Paper

1. Abstract
-provides an overview of the paper
2. Introduction
-provides a theoretical framework for the study
-provides an overview of what is already known
-clearly states the question and why it is important
3. Materials and Methods
-provides details of the experimental design
-provides details about how the data were collected and analyzed (including statistical
analysis)
4. Results
-provides a textual description of the results of analyses
-provides tables and/or graphs showing quantitative and statistical results of analyses
5. Discussion
-compares the results to what was previously reported in the primary literature -points out how the results either strengthen or weaken current theoretical models -if appropriate, makes suggestions on how theoretical models should be modified -highlights questions in need of further research
6. Literature Cited
-contains the full citation for every paper cited in the text. Does not contain citations that are not cited in the text

As you locate an example of each graph in the literature, download a digital copy, insert into graphexercise, and save in order - Fig. 11 Histogram, Fig. 12 Bar, Fig. 13 Dot, Fig. 14 Box, and Fig. 15 Scatterplot. Make sure to include the caption. Under each graph caption, type the citation of the paper where you found the graph. Proper citation format is: last name, initials, initials, last name, and initials, last name. year. title. journal volume:pages. Here's an example;

Harless, M.L., A.D. Walde, and D.K. Delaney. 2010. Sampling considerations for improving home range estimates of desert tortoises: effects of estimator, sampling regime, and sex. Herpetological Conservation and Biology 5:374-387.

Note: Histogram, Bar, Dot, Box, and Scatterplot are names given to particular graphs by SYSTAT. You may find different names in other statistical software and in the literature; for example, a histogram may be called a frequency distribution or a bar graph. Don't let that confuse you! You should be skilled enough to quickly determine the type of graph just by looking and applying your knowledge. For example, ask yourself what statistic is plotted on the graph; is it frequencies, means, medians, or individual cases?

Turn in a printed copy of graphexercise on the due date. Print two graphs per page. Do not separate the graphs from their respective captions.

## How to Search Primary Literature (Google Scholar; Library)



## Inferential Statistics <br> Home

## The Normal distribution

-very important frequency distribution for 2 reasons:
A. Data that are influenced by many small and unrelated random effects are approximately normally distributed (math: Fuzzy Central Limit Theorem); extremely widespread and common in nature
B. Forms the conceptual basis of a large number of statistical procedures - one of the most important theoretical distributions in statistics
C. Properties

1. formula: $\mathbf{1} /(\sigma \sqrt{ } 2 \pi) \exp \left(-(x-\mu)^{2} / 2 \sigma^{2}\right)$
2. students - no need to memorize the formula but you must be able to recognize it
3. shape determined by mean and SD
4. symetrical around the mean (mean=mode=median)

5. $\bar{x} \pm 1$ SD $=$ approx. $68 \%$ of cases; $\pm 2$ SD $=$ approx $95 \%$

D. Standard normal distribution
6. many different "normal" distributions
7. standardize any normal distribution (directly compare)
8. express individual cases in terms of $\mathrm{SND} ; \mathrm{z}=(\mathrm{x}-\overline{\mathrm{x}}) / \mathrm{s}$; "z-score"

9. z -score $=$ distance from mean in standard deviation units; e.g., $\mathrm{z}=1$ (=1SD greater than the mean)
10. Areas of normal curve (Tables)

E. Testing observed data for normality; SYSTAT output (TREAT.SYZ, EGGWGT)
11. qualitative: Probability plot (Graph $\rightarrow$ Distribution Plots $\rightarrow$ Probability Plot): DEMO
12. quantitative: Kolmogorov-Smirnov Test: DEMO
13. SYSTAT path: Analyze $\rightarrow$ Nonparametric Tests $\rightarrow$ One-sample KS (Enter selected variable and Lilliefors distribution)

- hypothesis: frequency distribution of EGGWGT is normally distributed
- test statistic, probability
- if probability $\leq 0.05$, reject the hypothesis; conclusion: EGGWGT distribution is not normally distributed (="skewed")
- if probability $>0.05$, cannot reject the hypothesis; conclusion: EGGWGT distribution is normally distributed

Exercise: practice SYSTAT Probability Plot and One-sample KS Test using the variable H2OOUT from file DLWMEANS.SYZ. Note that H2OOUT is not normally distributed (skewed)
-Data transformation has the potential to normalize non-normal data)

1. Data transformations - many procedures in statistics assume that data are normally distributed. If data are not normally distributed, one can transform the data to another measurement scale in an effort to normalize them. Deciding which transformation to use is entirely practical, i.e., the "right" transformation is whatever makes the data normally distributed. Trial-and-error applications of various transformations may be necessary to determine which will work. However, some transformations work better in some situations than in others. Examples of transformations commonly used in biology are the logarithmic, arcsine, and square-root transformations.

- the logarithmic transformation is useful in a wide variety of situations and is by far the most commonly used transformation in biology
- the arcsine (inverse sine) transformation is used specifically when data are in the form of proportions or percentages
- the square-root transformation is used specifically when data are in the form of counts

2. Transform the variable H2OOUT with common logarithms and retest for normality with both Probability Plot and KS. Note that the SYSTAT designation for common logs is L10 (always use common logs in Biol. 254). After transformation, the new variable L10H2OOUT should now be normal

## Always create a NEW variable name for the transformed variable!

Statistical inference - draw conclusions regarding populations based on analysis of samples from those populations

| Population <br> (numerical properties= parameters) | "Error" | $\underset{\text { (numerical }}{\frac{\text { Sample }}{\text { properties }}=\text { statistics) }}$ |
| :---: | :---: | :---: |
| The population is what we want to understand. |  | The sample is what we use to understand the population. <br> - Descriptive stats: describe data in the sample <br> - Inferential stats: infer from sample to population |

1. Two major categories of statistical inference
a. Estimate parameters (e.g., $\mu, \sigma$ )
b. Test hypotheses (infer population from sample)
2. The foundation for both concepts is the Sampling Distributioin
a. take repeated samples from population
b. examine distribution of sample means
3. Two major predictions of the Central Limit Theorem regarding sampling distributions
a. Means of samples from a normally distributed population will be normally distributed

- mean of means $=\Sigma \bar{x} / n$
- $\operatorname{SD}$ of means (=standard error of mean, SE or SEM ); $\mathrm{SE}=\mathrm{SD} / \sqrt{ } \mathrm{n}$
b. Means of samples from a non-normally distributed population will be normally distributed if $\mathbf{n}$ is sufficiently large (required $n$ is proportional to amount of variation)


## Simulation: Rice University Virtual Stats Lab

## Estimation of parameters

1. How well does the sample mean ( $\bar{x}$ ) estimate the population mean ( $\mu$ )?
a. in a normally distributed population, $95 \%$ of the cases lie between $\bar{x}-1.96$ SD and $\bar{x}$ +1.96 SD
b. in a normal sampling distribution, $95 \%$ of the means lie between $\overline{\mathrm{x}}-1.96$ SE and $\overline{\mathrm{x}}$ $+1.96 \mathrm{SE}$
c. interpretation: $\mathbf{9 5 \%}$ chance that population mean is enclosed within these limits ( $\mathbf{9 5 \%}$ confidence limits)

Absolute certainty is a privilege of uneducated minds-and fanatics. It is, for scientific folk, an unattainable ideal. - C. J. Keyser
d. problem: sampling distributions of means may depart from normality if sample size is small (central limit theorem)
e. solution: use distribution that adjusts for sample size - Student's t-distribution (shape determined by 3 characteristics):

- mean, SD, df
- areas of curve that exclude a given proportion of the distribution vary with n (Tables)
- at infinity df, $\mathrm{t}_{0.05}=$ 1.96 as in normal distribution

f. to calculate $95 \%$ CLs using a t-distribution, replace 1.96 with value from $t$-table
- UL: mean $\left.+\left(\mathrm{t}_{[0.05}, \mathrm{n}-1\right]\right) \times \mathrm{SE}$
- LL: mean - $\left.\left(\mathrm{t}_{[0.05}, \mathrm{n}-1\right]\right) \times$ XE
g. examples: calculate $95 \%$ CLs for these sample means:
- $\mathrm{x}=4.7, \mathrm{SD}=0.27, \mathrm{~N}=25$
95\% CI = 4.58-4.81 (higher n; narrower CLs)
- $\bar{x}=4.7, \mathrm{SD}=0.27, \mathrm{~N}=7$
$\underline{95 \%} \mathrm{CI}=\mathbf{4 . 4 5} \mathbf{- 4 . 9 5}$ (lower n ; broader CLs)

2. $95 \% \mathrm{CL}$ in the public media: GPS accuracy, political polls, church surveys
3. How to report sample means

- $\bar{x} \pm$ SD - provides idea of how much variation there is in the data but does not provide information on how well statistic $\overline{\mathrm{x}}$ estimates parameter $\mu$
- $\quad \bar{x}(95 \%$ CLs $)$ provides information on how well $\bar{x}$ estimates $\mu$ and if two means are significantly different from each other
- $\bar{x} \pm 1 S E$ (most common way of reporting means in text, tables, and graphs)



## Differences in means: graphic methods for 'informed guessing' whether means are statistically different



Caution! The most common way of reporting descriptive statistics in the literature is mean $\pm 1 \mathrm{SE}$; Proper interpretation requires that you visually double the value of the SE to get $95 \%$ CLs.


To properly interpret graphs displaying descriptive statistics, you must know what the error bars represent! (info found in the figure caption or in the M\&M)

## II. Hypothesis testing

A. Scientific hypothesis testing (sci_method)

1. Scientific method (ID; science begins when we try to explain observations (hypothesis)
2. Primary attributes of a good hypothesis
a. if it is correct, then it will explain what has been observed (consistent with observations)
b. if it is false, it can be shown to be false (falsifiable)
3. Cannot prove a true hypothesis; science advances by disproving false hypotheses
4. Process of hypothesis testing
a. if-then logic (IF the hypothesis is true, THEN this should be the result); MP?
b. if testing results in something other than expected outcome, we reject hypothesis and look for a better explanation
B. Statistical hypothesis testing - similar procedure
5. State hypothesis such that there are only 2 possible outcomes, e.g.,
a. $\quad \mathrm{H}_{\mathrm{A}}: \mathrm{A} \neq \mathrm{B}$ (cannot test directly) $=$ research [alternative] hypothesis
b. $\quad \mathrm{H}_{0}: \mathrm{A}=\mathrm{B}$ (if false; assume $\mathrm{H}_{\mathrm{A}}$ by default $)=$ null hypothesis
6. Example 1: compare case with known population $\mathrm{H}_{0}$ : case is from population $\mathrm{H}_{\mathrm{A}}$ : case is not from population What is the probability that the null hypothesis is true? -if low, the research hypothesis is more likely true

7. Example 2: compare sample mean with known population

- SYSTAT: Analyze->Hypothesis Testing->Mean->one-sample $t$-test
a. SYSTAT (onesamplet.syz): $\overline{\mathrm{x}}=598 ; \mathrm{SD}=70.3$; $\mathrm{n}=30$
b. assume population mean is known $[\mu=568]$
c. $\mathrm{H}_{0}: \overline{\mathrm{x}}=\mu ; \mathrm{H}_{\mathrm{A}}: \overline{\mathrm{x}} \neq \mu$

d. calculate (SYSTAT); one-sample t -test; test statistic, $\mathrm{t}_{\text {calc }}=2.31$
e. determine probability by comparing $\mathrm{t}_{\text {calc }}$ to $\mathrm{t}_{\text {tab }}$ (tabled value; $\mathrm{df}=29$; Tables); $\mathrm{P}=$ between 0.02 and 0.05 )
f. at $\mathbf{P}=\mathbf{0 . 0 5}$ (alpha level); $\mathrm{t}_{\text {tab }}=\mathbf{2 . 0 4 5}$ (critical value)
g. $\mathrm{t}_{\text {calc }}(2.31)$ is greater than $\mathrm{t}_{\text {tab }}(2.045)$, therefore $\mathrm{P}<0.05$
h. two explanations for obtaining a high $t$ value (2.31)
- null hypothesis is true; sample mean differed by chance alone (unlikely)
- null hypothesis is false (more likely)
i. 1-sample t-test: rarely done in science... Why?

4. Example 3: compare two sample means
(populations unknown - common question in many areas of biology)

- Hypothesis Testing 1 (next lecture section)

C. Writing null hypotheses for parametric difference tests and their nonparametric counterparts (does not include tests of frequencies or tests of relationships): required components

1. indicator $\left(\mathrm{H}_{0}\right)$
2. parameter (e.g., $\mu, \sigma^{2}$ )
3. variable (e.g., length, mass)
4. group (e.g., sex, color); for questions of differences between independent data only (no grouping variable for dependent data)
5. relational operator (e.g., $=, \geq, \leq$ )
-groups are designated by being enclosed in parentheses
-examples: independent: $\mathrm{H}_{0}: \mu$ length(males) $=\mu$ length(females)
dependent: $\mathrm{H}_{0}: \mu$ beforelength $=\mu$ afterlength
D. Two-tailed vs. one-tailed hypotheses
6. two-tailed research hypothesis: $\mathbf{H}_{\mathrm{A}}: \mu_{\mathrm{A}} \neq \mu_{\mathrm{B}}$ (non-directional) -null hypothesis (opposite of $\mathrm{H}_{\mathrm{A}}$ :): $\mathbf{H}_{\mathbf{0}}: \mu_{\mathrm{A}}=\boldsymbol{\mu}_{\mathrm{B}}$
7. one-tailed research hypothesis: $\mathbf{H}_{\mathrm{A}}: \mu_{\mathrm{A}}<\mu_{\mathrm{B}}$ (directional)
-null hypothesis (opposite of $\mathrm{H}_{\mathrm{A}}:$ ): $\mathbf{H}_{\mathbf{0}}: \mu_{\mathrm{A}} \geq \mu_{\mathrm{B}}$
8. one-tail: use only $1 / 2$ of distribution (divide probability by 2 )
9. how know if one-tail or two-tail? read question carefully

## III. Statistical decision-making

1. researchers set alpha level before statistical test is performed (usually 0.05 )
2. onesample.syz example: what would happen if you changed alpha to 0.01 after the test was done? (Tables; $\mathrm{t}_{\text {calc }}(2.31)<\mathrm{t}_{\text {tab }}(2.756 ; \mathrm{P}>0.01)$
3. possible to reject or not reject null hypothesis with the same set of data! Which one is "true?" (two types of errors)

| TRUTH TABLE | The real world; $\mathrm{H}_{0}$ is actually: |  |  |
| :--- | :--- | :--- | :--- |
|  |  | TRUE | FALSE |
| Your analysis; you say <br> that $\mathrm{H}_{0}$ is: | true | Correct | Type II error |
|  | false | Type I error | Correct |

- type I error (rejecting a true null hypothesis); fixed value set by scientific community ( $\mathrm{P}=0.05$ ); make mistake 1 out of 20 times
- type II error (failure to reject a false null hypothesis); can be minimized by:

1. increasing sample size
2. choosing the most powerful test (power $=$ probability of rejecting a false null hypothesis); minimum power of $80 \%$ generally necessary for an acceptable biological conclusion when you cannot reject the null hypothesis

- Why not reduce probability of type I error? - increases probability of type II error
- Alpha set at 0.05 because it represents a compromise between making type I and type II errors
- SYSTAT - how to calculate power or to determine minimum sample size needed for a specific power level (Utilities->Power Analysis->specific test)


## 4. Medical application of Truth Table - Diagnostic Testing Outcomes

|  | $\frac{\text { Disease present }}{\text { (pregnant) }}$ | $\frac{\text { Disease absent }}{\text { (not pregnant) }}$ |
| :---: | :---: | :---: |
| Test positive (cannot reject $\mathrm{H}_{\mathbf{0}}$; pregnant) | True positive | False positive (Type II error) |
| Test negative <br> (reject $\mathrm{H}_{0}$; not pregnant) | False negative (Type I error) | True negative |



a. conventional method (non-exact probability from statistical table)

- nonsignificant $=\mathrm{P}>0.05=\mathrm{ns}$
- significant $=\mathrm{P} \leq 0.05=$ *
- highly significant $=\mathrm{P} \leq 0.01=* *$

Statistical decisions are always $\underline{\text { made at the } P \leq 0.05 \text { level. }}$

- very highly significant $=\mathrm{P} \leq 0.001=$ *** $^{*}$

Absolute certainty is a privilege of uneducated minds-and fanatics. It is, for scientific folk, an unattainable ideal. - C. J. Keyser
b. modern method (exact probability from computer calculation)
c. both methods are correct, so students may use either method in Biol. 254
d. "Statistically significant" is one of those phrases scientists would love to have a chance to take back and rename. "Significant" suggests importance; but the test of statistical significance, developed by the British statistician R.A. Fisher, doesn't measure the importance or size of an effect; only whether we are able to distinguish it, using our keenest statistical tools, from zero. "Statistically noticeable" or statistically discernable" would be much better." -Mathematician Jordan Ellenberg
e. if you are talking science, avoid using the non-qualified term "significant" in a nonstatistical context
5. Why is it incorrect to "accept" a null hypothesis?
a. it implies that the null hypothesis has been proven true ( NO !); the null hypothesis is only assumed true
b. legal analogy: defendant is assumed innocent until proven guilty (jury decisions:
"guilty" or "not guilty")
c. modern experimental design was developed by Ronald Fisher (1930s). "...it should be noted that the null hypothesis is never proved or established, but is possibly disproved in the course of experimentation."
IV. Statistical Software (usually found toward the end of $M \& M$ in primary literature papers)

- SAS (no. 1 statistical software for scientists); high learning curve
- SYSTAT
- Minitab
- SPSS
- many others (http://en.wikipedia.org/wiki/Comparison_of_statistical_packages)
- Excel is not recommended for inferential statistical analysis.

| $\frac{\text { STATISTICAL TESTS }}{\text { Home }}$ |  | Parametric -more power, more assumptions | Nonparametric -less power, fewer assumptions | Assumptions of parametric <br> tests <br> - $\quad$ Data are randomly |
| :---: | :---: | :---: | :---: | :---: |
| Differences | Frequencies | ----- | Goodness-of-fit (GOF) <br> - Chi-square <br> - KolmogorovSmirnov (KS) | sampled and independent (except dependent designed tests( = repeated measures) <br> - Data are measured on ratio or interval scale |
|  | Variances | Bartlett's | Levene's | - Data (or residuals in ANOVA and regression) |
|  | 2 Means | $t$ - tests <br> - Independent samples t <br> - Paired samples t (assumes data are dependent) | Mann-Whitney <br> Wilcoxon (assumes data are dependent) | are normally distributed for each group <br> - For questions regarding means, the variances among groups (or residuals in ANOVA and regression) are |
|  | >2 Means | Analysis of Variance <br> - One-way ANOVA <br> - Post-hoc pairwise comparisons (Tukey) <br> - Two-way ANOVA | Kruskal-Wallis <br> Post-hoc pairwise comparisons (Dwass-Steel-CritchlowFligner; DSCF) | homogeneous <br> Assumptions of nonparametric tests <br> - Data are randomly sampled and independent (except dependent |
| Relationships | Frequencies | ----- | Test of Independence (contingency table analysis) <br> - Chi-square <br> - Fisher Exact Test |  |
|  | Variables/ Cases | Pearson correlation | Spearman correlation |  |
|  |  | Linear Regression | ----- |  |
| How does one know which test is appropriate? <br> - Read question carefully; make sure you understand what the question is asking <br> - Look for key words in the question: difference, differ, same as, more/less than, relationship, association, correlation, linked <br> - A " $v$ " word, (vary, variance, variation) will be present in the question for differences in variances <br> - If a "v" word does not appear in a difference question and question does not concern frequencies, assume question concerns means <br> - "Affect" and "effect" can be used in both difference and relationship questions. You must understand their use in context; for example, it likely is a difference question if there is a grouping variable present. |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

## Protocol for hypothesis testing - fill in each blank; write "NA" for questions that are not applicable.

 HomeA. Justify test used [2].

1. What are the variables? [.2] $\qquad$
2. What is the respective measurement scale of each variable? [.2] $\qquad$
3. Is the question about differences or relationships? [.2] $\qquad$
a. If a difference question, does it concern means, variances, or frequencies? [.2] $\qquad$
b. If a relationship question, does it concern variables or frequencies? [.2] $\qquad$
4. To determine if a parametric test can be used, ask these questions:
a. Means: If you think the appropriate test is a parametric test of differences in means -are the data independent or dependent? [.2] $\qquad$ -is each group/variable normally distributed? [.2] Y/N $\qquad$ ; probs $\qquad$ -are the variances homogeneous? [.2] Y/N $\qquad$ ; prob. $\qquad$
b. Variances: If you think the appropriate test is a parametric test of differences in variances, -is each group normally distributed? [.2] Y/N $\qquad$ ; probs. $\qquad$
c. Variables: If you think the appropriate test is a parametric test of relationships between variables, -are the residuals or each variable normally distributed? [.2] Y/N $\qquad$ ; probs $\qquad$
B. State research hypothesis(es) [0]. $\mathrm{H}_{\mathrm{A}}$ : $\qquad$
C. State null hypothesis(es) [2]. $\mathrm{H}_{0}$ :
(variables must match answers in A1)
D. What is the most appropriate test? [1]
(an incorrect answer limits further points)
E. Execute test(s) and identify and state value of each test statistic [2]. $\qquad$ (an incorrect answer limits further points)
F. State probability of each test statistic [1]. $\qquad$
G. State reject or cannot reject for each null hypothesis [1]. $\qquad$
H. Concisely state a biological conclusion for each test [1].

## Hypothesis Testing 1

## Frequencies: Goodness-of-Fit StatTests

1. Test whether an observed frequency distribution fits an expected frequency distribution
2. One variable, mutually exclusive categories, each frequency occurs in one category, no cell has an expected frequency <5 (must pool categories if violated), no proportions or percentages
3. Null hypothesis: $\mathbf{H}_{\mathbf{0}}:$ Ovar = Evar
4. Test statistic $\left(\chi^{2}\right)$ and probability source: Calculator/Statistical Table
-calculation: $\boldsymbol{\chi}^{\mathbf{2}}=\Sigma\left((\mathbf{O}-\mathbf{E})^{\mathbf{2}} / \mathbf{E}\right)$; reading a chi-square table (Tables)
5. Probability models used for determining expected frequencies

- The equal probability model occurs if all categories are equally likely. The expected number of outcomes for each category is $\mathrm{n} / \mathrm{no}$. categories.
- The unequal probability model occurs if there are several categories with unequal probabilities. The expected number of outcomes for each category is $n \mathrm{n} 1, \mathrm{np} 2$, ..etc.
- The binomial distribution model occurs if there are two possible outcomes for any item, with a constant probability of success with repeated independent encounters of subjects. To calculate the expected number of outcomes in $n$ experiments, multiply the binomial probabilities by $n$.
- The Poisson distribution model is used as a probability model for events that occur randomly. To calculate the expected number of outcomes in $n$ experiments, multiply the Poisson probabilities by n.

8. df: extrinsic hypothesis (theoretical): $\mathrm{df}=$ no. categories -1
df: intrinsic hypothesis (empirical; e.g., estimating the mean from the data): $\mathrm{df}=$ no. categories - 2
9. Examples:

- Question 1: Is the sex ratio of Wood Ducks skewed? (equal probability model; extrinsic)
- Question 2: Do Rough Green Snakes prefer a particular kind of tree when sleeping? (unequal probability model; intrinsic); pic)
- Question 3: Do the sample data fit a binomial distribution? (Binomial model; extrinsic; PP\#36)
- Question 4: Are seedlings randomly distributed among quadrats? (Poisson model; intrinsic; PP \#60)


## Example problems

1. Two purple-flowered pea plants, both heterozygous for flower color, were crossed, resulting in 78 purple-flowered offspring and 22 white-flowered offspring. Question: Does this outcome differ from the expected 3:1 ratio of purple-flowered to white-flowered offspring? (Protocol link)
2. The data below are number of juvenile manatees killed by boats in Florida. Question: Are males and females equally susceptible to being killed by boats? (Protocol link)
no. males killed (1985-1995): 206
no. females killed (1985-1995): 127

Frequencies: Test of Independence (=test of association) StatTests

1. Test whether the frequencies of two categorical variables are independent (unrelated)
2. Two categorical variables, each frequency occurs in multiple mutually exclusive categories, no proportions or percentages, no cell has an expected frequency of <5 (Systat will inform you of violations)
3. Null hypothesis: H0: row var independent of column var
4. Test statistic ( $\left.\mathbf{X}^{\mathbf{2}}\right)$ and probability source: Systat/Systat
5. SYSTAT path: Analyze $\rightarrow$ Tables $\rightarrow$ Two-Way (enter row and column variables)
6. Question: Is habitat dependent on (related to) sex?

| SYSTAT output: (GINMOVE.SYZ; Plummer) |  |  |  |
| :---: | :---: | :---: | :---: |
| Frequencies <br> HAB\$ (rows) by SEX\$ (columns) |  |  |  |
|  |  |  |  |
| F M | Total |  |  |
| +---------------- |  |  |  |
| P \| 480420 | | 900 |  |  |
| R\| 225 | | 27 |  |  |
| +---------------+ |  |  |  |
| Total 482445 | 927 |  |  |
| Test statistic | Value | DF | Prob |
| Pearson Chi-square | 22.1511 | 1.0000 | 0.000 |

6. Frequency table data - start with table (no raw data)
a. example 1 - Question: Is there an association between the hemoglobin $S$ allele and resistance to malaria?

|  | Did not <br> contract <br> malaria | Contracted <br> malaria |
| :--- | :--- | :--- |
| Heterozygotes | 1 | 14 |
| Homozygotes | 13 | 2 |


b. example 2 - Question: Is the frequency of breaking bones independent of taking calcium supplements? (supplements)

## Example problems

1. The following data are frequency of rabies in skunks collected from three geographic areas.

Question: Is the incidence of rabies dependent on geographic area? (Protocol link)

| Area | With <br> Rabies | Without <br> Rabies |
| :--- | :--- | :--- |
| Ozarks | 14 | 29 |
| Ouachitas | 12 | 38 |
| Delta | 11 | 35 |

2. The following data are frequency of individuals with different hair colors according to sex.

Question: Is human hair color dependent on sex? (Protocol link)

| sex | black | brown | blond | red |
| :--- | :--- | :--- | :--- | :--- |
| male | 32 | 43 | 16 | 9 |
| female | 55 | 65 | 64 | 16 |

## Frequencies: Fisher Exact Test StatTests

1. Test whether the frequencies of two categorical variables are independent; $\mathbf{2} \mathbf{x} \mathbf{2}$ table only
2. Two categorical variables, each frequency occurs in multiple mutually exclusive categories, no proportions or percentages; no minimum expected cell frequency
3. Null hypothesis: $\mathbf{H}_{\mathbf{0}}$ : row var independent of column var
4. Calculates probability directly; no intermediate test statistic
5. SYSTAT path: Analyze $\rightarrow$ Tables $\rightarrow$ Two-Way (check Fisher's Exact Test in Measures, enter row and column variables)
6. Question: Is phenotype independent of genotype?
Measures of Association for genetics\$ and malaria\$

| genetics(rows) by <br> malaria(columns) <br> n | $\mathbf{y}$ | Total |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| het | 14 | 1 | 15 |
| hom | 2 | 13 | 15 |
| Total | 16 | $\mathbf{1 4}$ | $\mathbf{3 0}$ |$\quad$| Test Statistic | Value | df | p-Value |
| :--- | :--- | :--- | :--- | :--- |

## Variances: Bartlett's and Levene's Tests StatTests

1. Test whether sample variances are from the same population (=homogeneous)
2. Bartlett's is sensitive to departures from normality (not robust)
3. Null hypothesis: $\mathbf{H}_{0}: \sigma^{2} \operatorname{var}($ group $\mathbf{a})=\sigma^{2} \operatorname{var}(\operatorname{group} \mathbf{b})=\sigma^{2} \operatorname{var}(\operatorname{group} \mathrm{c})$, etc.
4. Test statistic for Bartlett's test $\left(\chi^{2}\right)$ and Levene's test $(\mathbf{F})$ and probability source: Systat/Systat
5. SYSTAT path: Analyze $\rightarrow$ Hypothesis Testing $\rightarrow$ Variance $\rightarrow$ Equality of Several Variances (enter dependent and grouping variables)
6. Question: Does variation in total absorbance differ between concentrations?

| SYSTAT output: (ABSORBANCE.SYZ; Moore) |  |  |  |
| :---: | :---: | :---: | :---: |
| -Equality of Several Variances |  |  |  |
| Variable | CONC | N Mean Varian | nce Median |
| ABSORB_TOT | 8 | 6.0000 .4760 .020 | 0.467 |
|  |  | 6.0000 .4120 .052 | 0.449 |
| Bartlett's Test |  |  |  |
| Variable Chi-Square df p-Value |  |  |  |
| ABSORB_TOT1.004 |  | 1.0000 .316 |  |
| Levene's Test - *For Levene's, use the F-ratio based on the median. |  |  |  |
| Variable <br> ABSORB_TOT |  | F-Ratio | df p-Value |
|  | Based | on Mean 1.173 | 1,100.304 |
|  | Based | on Median 1.045 | 1,100.331 |

## Example problems

1. The following data are systolic blood pressure in two breeds of domestic cats. Question: Does variation in pressure $(\mathrm{mm} / \mathrm{Hg})$ differ between Siamese and Mynx cats? (Protocol link)

Siamese:122, 138, 129, 152, 149, 166, 110, 114, 155, 136, 189, 145, 129, 115, 144, 134
Mynx: $129,128,109,115,108,116,125,124,117,132,111,113,127$
2. Three different methods were used to determine the dissolved oxygen content of lake water. Each of the three methods was applied to a sample of water six times, with the following results. Question:
Do the three methods yielded equally variable results? (Protocol link)

| method 1 | method 2 | method 3 |
| :--- | :--- | :--- |
| 10.96 | 10.88 | 10.73 |
| 10.77 | 10.71 | 10.79 |
| 10.90 | 10.88 | 10.78 |
| 10.69 | 10.86 | 10.82 |
| 10.87 | 10.70 | 10.88 |
| 10.60 | 10.89 | 10.81 |

3. The following data are growth rate ( $\mathrm{g} / \mathrm{d}$ ) in newborn rats fed four different diets. Question: Is growth rate equally variable among diets? (Protocol link)
$\operatorname{diet} \mathrm{A}: 1.6,1.9,0.9,1.1,1.5,1.0,1.8,1.6 \quad \operatorname{diet} \mathrm{C}: 0.8,0.9,0.5,0.6,0.7,0.5,0.9,0.8$
$\operatorname{diet} \mathrm{B}: 2.5,2.0,2.8,2.6,2.6,2.9,1.9,2.1$
$\operatorname{diet} \mathrm{D}: 1.0,1.1,0.7,0.8,0.9,0.7,1.1,1.0$
4. The following data are number of moths caught during the night by four different trap types.

Question: Is there a difference in the variance of trap effectiveness? (Protocol link)
Trap type 1: 41, 34, 33, 36, 40, 25, 31, 37, 34, 30, 38
Trap type 2: $52,55,62,56,64,56,56,55$
Trap type 3: $25,33,34,37,41,34,40,36$
Trap type 4: 36, 41, 33, 28, 34, 40, 27, 37

## REVIEW

Graphic methods for 'informed guessing' whether means are statistically different (not a substitute for a formal statistical test)


Fig. 1. (A) Feces production ( $\times 1000$ ) in juvenile and adult green snakes by month. (B) Feces production (x1000) in adult male and female green snakes by month. Plotted are $\underline{\text { means } \pm 2 \text { SE. }}$

## VS.

Fig. 1. (A) Feces production ( $\times 1000$ ) in juvenile and adult green snakes by month. (B) Feces production (x1000) in adult male and female green snakes by month. Plotted are means $\pm 1$ SE.


What is the message of this image? Is there anything wrong with how it is portrayed?


## Means: Independent samples t-test StatTests

1. Test whether two sample means are from the same population
2. Powerful, robust (in literature ="Students" t-test"; William Gossett 1904)
3. Null hypothesis: $\mathbf{H}_{0}: \mu \operatorname{var}(\operatorname{group} \mathbf{a})=\mu \operatorname{var}(\operatorname{group} \mathbf{b})$
4. Test statistic ( $\mathbf{t}$; absolute value) and probability source: Systat/Systat
5. SYSTAT path: Analyze $\rightarrow$ Hypothesis Testing $\rightarrow$ Mean $\rightarrow$ Two-Sample $t$-test (enter dependent and grouping variables)
6. Calculate power if you cannot reject $\mathrm{H}_{0}$

Question: Do IAA levels differ between the wild type and triple mutants in the 4D germination treatments?

| a. | $H_{0}: \mu \mathrm{IAA}(\mathrm{WS})=\mu \mathrm{IAA}(\mathrm{ILR} / \mathrm{IAR} / \mathrm{ILL})$ |
| :--- | :--- |
| b. | pooled variance t ("regular" t -test - assumes homogeneous variances); use this one |
| c. | separate variance t ("approximate" t-test - does not assume homogeneous variances) |

SYSTAT output: (DRAMPEY.SYZ; Rampey)

| Variable | PLANT\$ | $\mathbf{N}$ | Mean | Standard <br> Deviation |
| :--- | :--- | :--- | :--- | :--- |
| IAA | ILR/IAR/ILL | 3.000 | 11.133 | 3.350 |
|  | WS | 3.000 | 21.000 | 3.378 |

## Separate Variance

| Variable | PLANT\$ | Mean Difference | 95.00\% Confidence Intervalt |  |  | df | p-Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Lower Limit | Upper Limit |  |  |  |
| IAA | ILR/IAR/ILL | -9.867 | -17.493 | -2.240 | -3.5924.000 |  | 0.023 |
|  | WS |  |  |  |  |  |  |


| Variable | PLANT\$ | Mean Difference | 95.00\% Confidence Interval |  |  | df | $p$-Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Lower Limit | Upper Limit |  |  |  |
| IAA | ILR/IAR/LLL | -9.867 | -17.493 | -2.241 | $-3.5924 .000$ |  | 0.023 |
|  | WS |  |  |  |  |  |  |  |

Example probleíms

1. The effect of copper sulfate on the mucus cells in the gill filaments of a species of fish was investigated. The number of mucus cells per square micron in the gill filaments of untreated fish and in fish exposed for 24 hours to copper sulfate ( $\mathrm{mg} / \mathrm{l}$ ) was as follows. Question: Does exposure to copper sulfate affect the number of mucus cells in these fish? (Protocol link)

$$
\begin{aligned}
& \text { untreated: } \\
& \text { exposed: } \\
& \text { ex, 17, } 12,18,11,18,12,15,16,14,18,12 \\
& 8,12,13,14,6,5,7,10,11,9,8
\end{aligned}
$$

2. A species of bacterium was grown with either glucose or sucrose as a carbon source. After a period of incubation, the number of cells ( $\mathrm{X} 10^{6}$ ) was determined. Question: Is there a difference in growth rate of the bacterium between the two carbon sources? (Protocol link)
glucose: $6.3,5.7,6.8,6.1,5.2$
sucrose: $5.8,6.2,6.0,5.1,5.8$

## Means: Mann-Whitney StatTests

1. Test whether two sample means are from the same population
2. Null hypothesis: $\mathbf{H}_{\mathbf{0}}: \boldsymbol{\mu v a r}$ (group a) $=\mu \mathbf{v a r}$ (group b) (technically testing differences in medians)
3. Test statistic (U) and probability source: Systat/Systat (if provided an outside answer, may need to convert test statistic $\left(U^{\prime}=\mathrm{n}_{1} \mathrm{n}_{2}-\mathrm{U}\right)$
4. SYSTAT path: Analyze $\rightarrow$ Nonparametric Tests $\rightarrow$ Kruskal-Wallis (enter dependent and grouping variables)
5. 
6. Question: Does weight differ between the sexes?
```
SYSTAT output: (LONOKE.SYZ; Plummer)
Mann-Whitney U Test for female length within range as male
<50) length.
The categorical values encountered during processing are
\begin{tabular}{|l|c|}
\hline Variables & Levels \\
\hline SEX (2 levels) 1.0002 .000 \\
\hline
\end{tabular}
\begin{tabular}{|l|l|}
\hline Dependent Variable & WGT \\
\hline Grouping Variable & SEX \\
\hline
\end{tabular}
\begin{tabular}{|l|l|l|}
\hline Group & Count & Rank Sum \\
\hline 1 & 27 & 912.000 \\
\hline 2 & 44 & \(1,644.000\) \\
\hline
\end{tabular}
    Mann-Whitney U Test Statistic : 534.000
    p-Value : 0.477
    Chi-Square Approximation : 0.505
    df : 1
```


## Example problems

1. Twenty people were randomly assigned to two groups of ten each. One group viewed a hairy spider, and the other group viewed a similar but nonhairy spider. Each person was asked to score the spider she or he viewed on a ranked scariness scale from 1 to 10 ( 10 being the most scary). The results are below. Question: Do people find hairy spiders scarier than nonhairy spiders? (Protocol link).
hairy: $\quad 10,8,7,9,9,10,9,9,5,8$
nonhairy: $7,6,8,6,1,5,4,5,6,3$
2. The mass (g) of random samples of adult male tuatara from two localities in New Zealand are given below. Question: Do animals from locality A differ in mean mass from locality B? (Protocol link) loc A: $510,773,840,505,765,780,235,790,440,435,815,460,690$
loc B: $650,600,600,575,452,320,660$

## Means: Paired samples $\boldsymbol{t}$-test StatTests

1. Test whether two sample means are from the same population
2. Each individual is measured twice or selected pairs are matched ("repeated measures"); more powerful than independent t-test (reduced error variance); robust; Exercise in Twins, NASA Twins
3. Data must be in an unstacked format
4. Null hypothesis: $\mathbf{H}_{\mathbf{0}}: \boldsymbol{\mu} \mathbf{v a r} \mathbf{1}=\boldsymbol{\mu} \mathbf{v a r} 2$ (no grouping variable)
5. Test statistic (t) and probability source: Systat/Systat
6. SYSTAT path: Analyze $\rightarrow$ Hypothesis Testing $\rightarrow$ Mean $\rightarrow$ Paired $t$-test (enter paired variables)
7. Calculate power if you cannot reject $\mathrm{H}_{0}$

Question: Does early field metabolic rate differ from late field metabolic rate?

| SYSTAT output: (DLWMEANS.SYZ; Plummer) |  |  |  |
| :---: | :---: | :---: | :---: |
| Paired samples $t$ test on EARLYFMR vs LATEFMR with 6 cases |  |  |  |
| Mean EARLYFMR | = | 0.1552 |  |
| Mean LATEFMR | = | 0.1268 |  |
| Mean Difference | = | 0.0283 | $95.00 \% \mathrm{Cl}=-0.0580$ to |
| 0.1147 |  |  |  |
| SD Difference = |  | 0.0823 | $\mathrm{t}=0.8437$ |
|  |  | DF $=5$ | Prob $=0.4373$ |

How to stack dependent data files for testing equality of variances

1. manual stacking (create grouping variable)
2. SYSTAT stacking (Data $\rightarrow$ Reshape $\rightarrow$ Stack)

## Example problems

1. Brucella abortus antibody titers ( $\mathrm{pfc} / 10^{6}$ cells) in 15 turkeys were measured before and after a period of stress. Question: Did stress decrease antibody titer in these turkeys? (Protocol link)

| turkey no.: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| before stress: | 20 | 18 | 19 | 18 | 17 | 14 | 17 | 10 | 13 | 16 | 20 | 17 | 16 | 19 | 8 |
| after stress: | 17 | 14 | 16 | 19 | 14 | 18 | 8 | 10 | 12 | 15 | 8 | 6 | 17 | 5 | 3 |

2. Male hoop snakes, upon encountering one another, may engage in a protracted ritualized combat behavior until one establishes himself as dominant over the other. Six males were tested in the presence of a female and again in the absence of a female. Whether each male was tested first with or without a female was randomly determined. The results in interaction time (min.) are below.
Question: Do these encounters last longer in the presence of a female? (Protocol link)

| snake no.: | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| w/o female: | 10 | 15 | 8 | 30 | 1 | 80 |
| w/ female: | 59 | 35 | 70 | 65 | 43 | 90 |

## Means: Wilcoxon (Chap. 9) StatTests

1. Test whether two sample means are from the same population
2. Each individual is measured twice or selected pairs are matched (repeated measures)
3. Data must be in an unstacked format
4. Null hypothesis: $\mathbf{H}_{\mathbf{0}}: \boldsymbol{\mu} \mathbf{v a r} \mathbf{1}=\boldsymbol{\mu} \mathbf{v a r} 2$ (no grouping variable)
5. Test statistic (Z) and probability source: Systat/Systat
6. SYSTAT path: Analyze $\rightarrow$ Nonparametric Tests $\rightarrow$ Wilcoxon (enter paired variables)
7. Question: Does field metabolic rate differ between early and late measurements?

| SYSTAT output: (DLWMEANS.SYZ; Plummer) |  |  |
| :---: | :---: | :---: |
| Wilcoxon Signed Ranks Test Results |  |  |
| Counts of differences (row variable greater than column) |  |  |
|  | EARLYFMR | LATEFMR |
| EARLYFMR | 0 | 4 |
| LATEFMR | 2 | 0 |
| $Z=$ (Sum of signed ranks)/square root(sum of squared ranks) |  |  |
|  | EARLYFMR | LATEFMR |
| EARLYFMR | 0.0 |  |
| LATEFMR | -0.3145 | 0.0 |
| Two-sided probabilities using normal approximation |  |  |
|  | EARLYFMR | LATEFMR |
| EARLYFMR | 1.0000 |  |
| LATEFMR | 0.7532 | 1.0000 |

## Example problems

1. The wattle thickness ( mm ) of 10 randomly selected chickens was measured before and after treatment with PHA. Question: Does treatment with PHA affect wattle thickness? (Protocol link)

| Chicken no. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| pretreatment | 1.05 | 1.01 | 0.78 | 0.98 | 0.81 | 0.95 | 1.00 | 0.83 | 0.78 | 1.05 |
| posttreatment | 3.48 | 5.02 | 5.37 | 5.45 | 5.37 | 3.92 | 6.54 | 3.42 | 3.72 | 3.25 |

2. Ten young men were asked to rate their feeling of well-being on a scale of 1 (worst) to 10 (best) before and after taking an experimental drug. Question: Does the drug increase a person's sense of well-being? (Protocol link)

| individual no.: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| before drug: | 5 | 8 | 2 | 7 | 5 | 2 | 9 | 3 | 9 | 6 |
| after drug: | 7 | 9 | 1 | 9 | 5 | 9 | 9 | 9 | 10 | 7 |

You are responsible for knowing how to work all the Practice Problems concerning differences in frequencies, association of frequencies, and differences in variances and two means (Goodness-of-Fit, Test of Independence, Fisher's Exact Test,, Bartlett's, Levene's, Independent Samples t-test, Paired Samples t-test, Mann-Whitney, Wilcoxon). Exam problems will be taken directly or modified from Example and Practice Problems.

## Hypothesis Testing 2 <br> Home

## Analysis of Variance StatTests

1. ANOVA - important part of experimental design (Fisher 1935); extremely common in the literature
2. Goal is to partition the sources of natural variability for any given system
-total variability $=$ source $1+$ source $2+$ source 3 , etc. (additive)
3. Also permits measurement of interaction (e.g., drug interaction); source 1 x source2 (not additive)
4. Many different ANOVA models; e.g.,

- One-way ANOVA (1 dependent variable, 1 independent variable)
- Two-way ANOVA (1 dependent variable, 2 independent variables)
- Analysis of Covariance; ANCOVA (1 dep, 1 indep, 1 covariate) - at end of course if enough time


## One-way ANOVA

1. Test whether sample means are from the same population
2. Powerful and robust
3. Null hypothesis: $\mathbf{H}_{0}: \mu \operatorname{var}($ group 1$)=\mu \operatorname{var}($ group 2$)=\mu \operatorname{var}(\operatorname{group} 3)$, etc.
4. Why not use multiple t -tests? - "The problem of multiple comparisons"


$$
4 \text { means }=30 \%
$$

$$
5 \text { means }=50 \%
$$

5. Partition total variation into between-group and within-group ("error") variation

- between group: variation due to being part of a certain group (treatment)
- error variation: all variation not due to being in that group

6. Calculate ratio of between-groups variance/within-groups variance (F-ratio; test statistic)

- F-ratio relatively large when treatment accounts for significant variation

7. Determine probability; compare F-ratio with F-distribution (shape determined by 2 separate dfs)

- numerator (no. treatments - 1)
- denominator (no. observations in all groups - no. groups)

8. Test statistic (F) and probability source: Systat/Systat
```
REVIEW: Required components of a null hypothesis for questions of differences in means or variances.
    1. Indicator (H0)
    2. Parameter (e.g., }\mu,\mp@subsup{\sigma}{}{2}
    3. Variable (e.g., length, mass)
    4. Group (e.g., sex, color); for questions of differences between independent data only (no grouping
        variable for dependent data). Groups are designated by being enclosed in parentheses.
    5. Relational operator (e.g., =, \geq, \leq)
Examples:
    -independent: H0: }\mu\mathrm{ length(males) = }\mu\mathrm{ length(females)
    -dependent: H0: }\mu\mathrm{ beforelength = }\mu\mathrm{ afterlength
```

9. SYSTAT path: Analyze $\rightarrow$ ANOVA $\rightarrow$ Estimate Model (enter dependent and grouping[=factor] variables) $\rightarrow$ Options (KS, Levene)
10. Calculate power if you cannot reject $\mathrm{H}_{0}$

| SYSTAT output: (TREAT.SYZ; Plummer); treat.ppt |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Categorical values encountered during processing are: CLUTNO (24 levels) <br> $1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,24,25$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| Dep Var: EGGWGT N: 245 Multiple R: $\mathbf{0 . 7 0 9 0}$ Squared multiple R: $\mathbf{0 . 5 0 2 7}$ |  |  |  |  |  | Normality and homogeneity assumptions are tested after ANOVA with the residuals (=difference between observed value and value predicted by the model) |
| Analysis of Variance <br> Source <br> Sum-of-Squares DF |  |  |  |  |  |  |
|  |  |  | Mean-Square | F-Ratio P |  |  |
| CLUTNO | 334.2372 | 23 | 14.5321 | 9.7115 | 0.000 |  |
| Error | 330.6987 | 221 | 1.4964 |  |  |  |

## Example Problems

1. Random samples of a certain species of zooplankton were collected from five lakes and their selenium content (ppm) was determined. Was there a difference among lakes with respect to selenium content? (Protocol link)
lake A: $23,30,28,32,35,27,30,32$
lake B: $34,42,39,40,38,41,40,39$
lake C: $15,18,12,10,8,16,20,19$
lake D: $18,15,9,12,10,17,10,12$
lake E: 25, 20, 22, 18, 30, 22, 20, 19
2. The following data are amount of food $(\mathrm{kg})$ consumed per day by adult deer at different times of the year. Test the null hypothesis that food consumption was the same for all the months tested. (Protocol link)

| February | May | August | November |
| :--- | :--- | :--- | :--- |
| 4.7 | 4.6 | 4.8 | 4.9 |
| 4.9 | 4.4 | 4.7 | 5.2 |
| 5.0 | 4.3 | 4.6 | 5.4 |
| 4.8 | 4.4 | 4.4 | 5.1 |
| 4.7 | 4.1 | 4.7 | 5.6 |
| 4.2 | 4.8 |  |  |
|  |  |  |  |

## After significant ANOVA: Which means are different from which other means?

Post hoc pairwise tests counteract the problem of maintaining an alpha level of 0.05 for multiple comparisons; many different post hoc tests

1. Example: Tukey test
2. Test statistic (Difference) and probability source: Systat/Systat
3. SYSTAT path: Analyze $\rightarrow$ ANOVA $\rightarrow$ Pairwise comparisons $\rightarrow$ Tukey (add group)
4. MYSTAT path: not available

## SYSTAT output: (TREAT.SYZ; Plummer, select clutno<6); treat.ppt <br> Categorical values encountered during processing are: <br> CLUTNO (5 levels) - 1, 2, 3, 4, 5

Dep Var: EGGWGT N: 63 Multiple R: 0.5851 Squared multiple R: 0.3423
Analysis of Variance

| Source | Sum-of-Squares | DF | Mean-Square | F-Ratio | P |
| :--- | :--- | :--- | :--- | :--- | :--- |
| CLUTNO | 166.0769 | 4 | 41.5192 | $\mathbf{7 . 5 4 7 1}$ | $\mathbf{0 . 0 0 0 1}$ |
| Error | 319.0783 | 58 | 5.5014 |  |  |

Post Hoc Test of EGGWGT
Using least squares means.
Tukey's Honestly-Significant-Difference Test
CLUTNO(i)CLUTNO(j) Difference ${ }^{\text {p-Value }} 95 \%$ Confidence Interval


## Example Problems

1. In a study of snake hibernation, fifteen pythons of similar size and age were randomly assigned to three groups. One group was treated with drug A, one group with drug B, and the third group was not treated. Their systolic blood pressure $(\mathrm{mmHg})$ was measured 24 hours after administration of the treatments. Do the drugs affect blood pressure? If so, do they have similar effects? (Protocol link)

$$
\text { control: } 130,135,132,128,130
$$

drug A: $118,120,125,119,121$
drug B: $105,110,98,106,105$
2. Fourteen hucksters were assigned at random to one of three experimental groups and fed a different diet for six months. Use the following data on huckster mass ( kg ) at the end of the experiment to determine if diet affected body size. Which diet produced the heaviest hucksters? (Protocol link)

| diet 1 | diet 2 | $\operatorname{diet} 3$ |
| :--- | :--- | :--- |
| 60.8 | 68.7 | 102.6 |
| 57.0 | 67.7 | 102.1 |
| 65.0 | 74.0 | 100.2 |
| 58.6 | 66.3 | 96.5 |
| 61.7 | 69.8 |  |

## Kruskal Wallis Test

1. Test whether three or more sample means are from the same population
2. Non-parametric counterpart to one-way ANOVA
3. Null hypothesis: $\mathbf{H}_{\mathbf{0}}: \mu \operatorname{var}($ group 1$)=$ $\mu \mathrm{var}($ group 2$)=\mu \operatorname{var}($ group 3$)$, etc.
4. Test statistic (H) and probability source: Systat/Systat
5. SYSTAT path:

Analyze $\rightarrow$ Nonparametric
tests $\rightarrow$ Kruskal-Wallis (enter dependent and grouping (=factor) variables)

| Dwass-Steel-Chritchlow-Fligner <br> Test for All Pairwise Comparisons |  |
| :--- | :--- | :--- |
| Group(i) Group(j) Statistic p-Value  <br> 1 2 7.8558 0.0000 <br> 1 3 1.2552 0.9745 <br> 1 4 9.8964 0.0000 <br> 1 5 5.8438 0.0007 <br> 1 6 6.1237 0.0003 <br> 1 7 6.5521 0.0001 <br> 2 3 -4.1468 0.0524 <br> 2 4 0.9094 0.9954 <br> 2 5 0.8282 0.9972 <br> etc.    |  |



- For post hoc pairwise comparisons after significant $K W$


## Dwass-Steel-Critchlow-Fligner Test (DSCF)

## Example Problems

1. Twenty-four freshwater clams were randomly assigned to four groups of six each. One group was placed in deionized water, one group was placed in a solution of 0.5 mM sodium sulfate, and one group was placed in a solution of 0.74 mM sodium chloride. At the end of a specified time period, blood potassium levels ( $\mu \mathrm{M} \mathrm{K}^{+}$) were determined. Did treatment affect blood potassium levels? (Protocol link)
pond water: $0.518,0.523,0.499,0.502,0.520,0.507$
deionized water: $0.308,0.385,0.301,0.390,0.307,0.371$
sodium sulfate: $0.393,0.415,0.351,0.390,0.385,0.397$
sodium chloride: $0.383,0.405,0.398,0.352,0.381,0.407$
2. An entomologist interested in the vertical distribution of a fly species collected the following data on numbers of flies (no. flies $/ \mathrm{m}^{3}$ ) from each of tree different vegetation layers. Use these data to test the hypothesis that fly abundance was the same in all three vegetation layers. (Protocol link)

| herbs | shrubs | trees |
| :--- | :--- | :--- |
| 14.0 | 8.4 | 6.9 |
| 12.1 | 5.1 | 7.3 |
| 5.6 | 5.5 | 5.8 |
| 6.2 | 6.6 | 4.1 |
| 12.2 | 6.3 | 5.4 |

Two-way ANOVA - factorial design; 2 independent variables (=factors)

1. Test whether sample means are from the same population; access interaction between independent variables
2. Powerful and robust; test assumptions with residuals
3. Test statistic (F) and probability source: Systat/Systat
4. Null hypotheses:

- $\mathbf{H}_{0}: \mu \operatorname{var}($ group1 $)=\mu \operatorname{var}(\operatorname{group} 2)=\mu \operatorname{var}($ group3 $)$, etc. (for each main effect)
- $\mathbf{H}_{0}:$ no interaction among factors (interaction $=$ the extent to which the effects of one factor differ according to the levels of another factor; synergism or antagonism)

5. SYSTAT path: Analyze $\rightarrow$ ANOVA $\rightarrow$ Estimate Model (enter dependent variable and $>1$ grouping variable)

## Example 1:

SYSTAT output; MOUSEDIET.SYZ; Cooper

| Variables | Levels |
| :--- | :---: |
| DIET\$ (4 levels) | $5 \mathrm{~K}-96: A \mathrm{~A} N-\mathrm{cas}$ AIN-spi:P5001 |
| NPDOSE\$ (2 levels) | 0 |



## Conclusions

- Diet explains a significant amount of variation in body weight. Body weight is greater in mice with the P 5001 diet.
- NPdose explains a significant amount of variation in body weight. Body weight is greater in mice not receiving NPdose.
- There is no interaction between diet and NPdose. Body weight responds the same to diet and NPdose.


Example 2: effect of diet and stress on weight gain in mice


## How affect? Conclusions

- Diet explains a significant amount of variation in weight gain. Mice with junk food diets gain more weight than mice with regular diets.
- Stress explains a significant amount of variation in weight gain. Mice experiencing high stress gain more weight than mice experiencing low stress.
- The interaction between diet and stress explains significant variation in weight gain. Weight gain caused by a junk food diet is exacerbated (i.e. made worse) by high stress. Or stated from another perspective, the weight gain caused by high stress is exacerbated by a junk food diet.


## Example Problems

1. Use USOPHEO.SYZ; Plummer to determine if body size is affected by sex and/or location. Read the description of the data file before proceeding. (Protocol link)
2. Qualime epithelial cancer is hypothesized to result from either genotype or several environmental factors that vary by season. To address this hypothesis, use the data below on QSA level ( $\mu \mathrm{g} / \mathrm{g}$; the diagnostic test indicator of qualime cancer) that were collected on 20 individuals in different seasons. (Protocol link)

| QSA | Genotype | Season | QSA | Genotype | Season | QSA | Genotype | Season | QSA | Genotype | Season |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 478 | ZZ | Winter | 425 | ZW | Summer | 428 | ZZ | Summer | 466 | ZW | Winter |
| 538 | ZZ | Winter | 467 | ZW | Summer | 478 | ZZ | Summer | 522 | ZW | Winter |
| 502 | ZZ | Winter | 444 | ZW | Summer | 455 | ZZ | Summer | 489 | ZW | Winter |
| 496 | ZZ | Winter | 438 | ZW | Summer | 446 | ZZ | Summer | 475 | ZW | Winter |
| 483 | ZZ | Winter | 431 | ZW | Summer | 432 | ZZ | Summer | 501 | ZW | Winter |

3. Work practice problem \#56. Why is it a one-way rather than a two-way ANOVA? You will have to create a derived variable to work the problem. There are two ways to do this: (1) enter the derived variable directly on the SYSTAT data sheet or
(2) enter all of the data shown and use TRANSFORM $\rightarrow I f$.., Then Let to create the derived variable. You likely will need to review how to create derived variables.

## Correlation

- correlation analysis is a test of association that makes no assumption about a cause-and-effect relationship (i.e., there is no dependent and independent variable)
- addresses two questions
- does an association exist between two variables?
- if the association exits, what is its strength (effect)?
- requires that both variables be normally distributed random variables


## Pearson correlation StatTests

1. Test whether the cases of two variables are correlated (positive or negative)
2. Linear relationships only
3. Null hypothesis: assume no relationship; $\mathbf{H}_{\mathbf{0}}: \rho \operatorname{var} \mathbf{1}, \mathbf{v a r} \mathbf{2}=\mathbf{0}$ (Note there is no grouping variable, just two ratio or interval variables)
4. Test statistic (correlation coefficient, $\mathbf{r}$ (varies from -1 to +1 ; measure of strength) and probability source: Systat/Systat
5. $r^{2}$ (coefficient of determination) - proportion of variation in one variable that is explained by variation in the other variable ( $\mathrm{r}^{2}$ is not a test statistic)
6. SYSTAT path: Analyze $\rightarrow$ Correlation $\rightarrow$ Simple (enter variables; Continuous Data)
7. Calculate power if you cannot reject $\mathrm{H}_{0}$

| SYSTAT output (AMPHIBIANS.SYS; Mills); assume normality for purposes of demonstration only |  |  |
| :---: | :---: | :---: |
| Number of Non-Missing Cases: 40 |  |  |
| Means BUFO SPECIES |  |  |
|  |  |  |
| 1.57502 .5000 |  |  |
| Pearson Correlation Matrix |  |  |
| BUFO SPECIES |  |  |
| BUFO 1.0000 |  |  |
| SPECIES 0.61981 .0000 |  |  |
| Matrix of Bonferroni Probabilities |  |  |
|  |  |  |
| BUFO | 0.0000 |  |
| SPECIES | S 0.0000 | 0.0000 |

Bonferroni probability correction (counteracts the "The problem of multiple comparisons"); reduces chances of making a Type 1 error (= "false negative" in the medical literature)
SYSTAT output: (AMPHIBIANS.SYZ; Mills); as
demonstration only
Number of Non-Missing Cases: 40

| Means <br> BUFO |  |
| :--- | :--- |
| 1.57501 .77500 .75005 .1750 | 2.5000 |


| Pearson Corr | relation <br> BUFO | Matrix <br> RASP | HYLA | INDIVIDUAL | SPECIES |
| :---: | :---: | :---: | :---: | :---: | :---: |
| BUFO | 1.0000 |  |  |  |  |
| RASP | 0.2408 | 1.0000 |  |  |  |
| HYLA | 0.1034 | 0.2000 | 1.0000 |  |  |
| INDIVIDUALS | 0.7103 | 0.7239 | 0.5245 | 1.0000 |  |
| SPECIES | 0.6198 | 0.5630 | 0.4854 | 0.8761 | 1.0000 |


| Matrix of Bonferroni Probabilities <br>  <br>  <br> BUFO |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| RASP | HYLA | INDIVIDUALS | SPECIES |  |
| BUFO | 0.0000 |  |  |  |
| RASP | 1.0000 | 0.0000 |  |  |
| HYLA | 1.0000 | 1.0000 | 0.0000 |  |
| INDIVIDUALS | 0.0000 | 0.0000 | 0.0051 | 0.0000 |
| SPECIES | 0.0002 | 0.0016 | 0.0150 | 0.0000 |

Example problems

1. Use the following data on wing length $(\mathrm{cm})$ and tail length $(\mathrm{cm})$ in cowbirds to determine if there is a relationship between the two variables. (Protocol link)

| Wing | 10.4 | 10.8 | 11.1 | 10.2 | 10.3 | 10.2 | 10.7 | 10.45 | 10.8 | 11.2 | 10.6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Tail | 7.4 | 7.6 | 7.9 | 7.2 | 7.4 | 7.1 | 7.4 | 7.2 | 7.8 | 7.7 | 7.8 |

2. Use the following data taken from crabs to determine if there is a relationship between weight of gills (g) and weight of body $(\mathrm{g})$ and between weight of thoracic shield $(\mathrm{g})$ and weight of body. (Protocol link)

| Body | 159 | 179 | 100 | 45 | 384 | 230 | 100 | 320 | 80 | 220 | 320 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Gill | 14.4 | 15.2 | 11.3 | 2.5 | 22.7 | 14.9 | 11.4 | 15.81 | 4.19 | 15.39 | 17.25 |
| Thorax | 80.5 | 85.2 | 49.9 | 21.1 | 195.3 | 111.5 | 56.6 | 156.1 | 39.0 | 108.91 | 160.1 |

## Spearman correlation StatTests

1. Test whether the cases of two variables are correlated
2. Linear relationships only
3. Null hypothesis: $\mathbf{H}_{\mathbf{0}}: \boldsymbol{\rho}_{\mathrm{s}} \mathbf{v a r} 1, \operatorname{var} \mathbf{2}=\mathbf{0}$ (Note there is no grouping variable, just two ratio, interval, or ranked variables)
4. Test statistic ( $\mathbf{r}_{\mathrm{s}}$ ) and probability source: Systat/Statistical Table
5. SYSTAT path: Analyze $\rightarrow$ Correlation $\rightarrow$ Simple (enter variables; Rank Order Data)

| SYSTAT output: (AMPHIBIANS.SYZ; Mills) <br> Number of Non-Missing Cases: 40 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Spearman Co | rrelation BUFO | n Matrix RASP | HYLA | GACA | NOVI | INDIVIDUALS | SPECIES |  |  |
| BUFO | 1.0000 |  |  |  |  |  |  | Spearman probabilities are not available in SYSTAT; must get probabilities from a Spearman Table |  |
| RASP | 0.3113 | 1.0000 |  |  |  |  |  |  |  |
| HYLA | 0.2886 | 0.3879 | 1.0000 |  |  |  |  |  |  |
| GACA | 0.3407 | 0.3682 | 0.1901 | 1.0000 |  |  |  |  |  |
| NOVI | 0.2314 | 0.0436 | 0.3467 | -0.0526 | 1.0000 |  |  |  |  |
| INDIVIDUALS | 0.7264 | 0.7678 | 0.5804 | 0.3506 | 0.2044 | 1.0000 |  |  |  |
| SPECIES | 0.7512 | 0.6482 | 0.6438 | 0.3226 | 0.3001 | 0.9173 | 1.0000 |  |  |

Example problems

1. The following data are ranked scores for ten students who took both a math and a biology aptitude examination. Is there a relationship between math and biology aptitude scores for these students? (Protocol link)

| Math | 53 | 45 | 72 | 78 | 53 | 63 | 86 | 98 | 59 | 71 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Biology | 83 | 37 | 41 | 84 | 56 | 85 | 77 | 87 | 70 | 59 |

2. Test the following data to determine if there is a relationship between the total length of aphid stem mothers and the mean thorax length of their parthenogenetic offspring. (Protocol link)

| Mother | 8.7 | 8.5 | 9.4 | 10.0 | 6.3 | 7.8 | 11.9 | 6.5 | 6.6 | 10.6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| offspring | 5.95 | 5.65 | 6.00 | 5.70 | 4.40 | 5.53 | 6.00 | 4.18 | 6.15 | 5.93 |

## Correlation vs. causation

1. Earlier: alcoholics in FL vs HU grads; spurious correlations
2. sometimes results from a common correlation with 3rd variable (e.g., $B$ correlated with $C$ because both B\&C are functionally correlated with A); Cause and effect

Regression analysis is a test of association that

- assumes a cause-and-effect relationship between an independent and dependent variable
- is used to address the same basic questions as correlation analysis (with one important additional question), but from the perspective of cause-and-effect
- does the independent variable explain significant variation in the dependent variable?
- how strong is the explanatory power of the independent variable?
- what is the mathematical relationship between the variables? (i.e., what is the mathematical equation that describes the relationship?)
- requires that the dependent variable be a normally distributed random variable. The independent variable may be controlled or selected and thus may not be a normally distributed random variable.


## Regression (Chap. 14) StatTests

1. Test whether the cases of one variable are functionally (mathematically) related to the cases of another variable (i.e., can be predicted from)
2. Linear relationships only
3. Normality assumptions are analyzed with residuals after the regression analysis; robust
4. Null hypothesis: H0: $\beta \mathbf{y v a r}$, xvar $=\mathbf{0}$ (Note there is no grouping variable, just two ratio or interval variables)
5. Test statistic (F-ratio) and probability source: Systat/Systat
6. SYSTAT path: Analyze $\rightarrow$ Regression $\rightarrow$ Linear $\rightarrow$ Least Squares (enter dependent and independent variables; enter KS on options tab)

## Procedure

a. Fit regression line (least squares method; minimize $\Sigma\left(\right.$ residuals $^{2}$ )
b. Test for significance of slope
c. Write the regression equation (general form $\mathbf{Y}=\mathbf{a}$ (intercept) $+\mathbf{b}$ (slope) $\mathbf{X}$ -do NOT use math format ( $\mathrm{y}=\mathrm{mx}+\mathrm{b}$ )
d. Add regression statistics and variable names

## SYSTAT output: (SHRIMP.SYZ; Goy)

Output format

- Regression statistics: intercept (=constant), slope (=regression coefficient); standard error
- ANOVA table (test statistic, probability)
- KS test of assumptions

| Dependent Variable | EGGNO |
| :--- | :--- |
| N | 68 |
| Multiple R | 0.7763 |
| Squared Multiple R | 0.6027 |
| Adjusted Squared Multiple R | 0.5967 |
| Standard Error of Estimate | 1142.1881 |


| Regression Coefficients $\mathrm{B}=\left(\mathrm{X}^{\prime} \mathrm{X}\right)^{-1} \mathrm{X}^{\prime} \mathrm{Y}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Effect | Coefficient intercep | Standard Error | Std. Coefficient | Tolerance |  | p-Value |
| CONSTANT | -4914.5822 | 683.9281 | 0.0000 |  | -7.1858 | 0.0000 |
| FEMLEN | 561.5867 | 56.1225 | 0.7763 | 1.0000 | 10.0065 | 0.0000 |


| Analysis of Variance |  |  |  |  | $\leftarrow$ Test statistic and probability |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | \|SS df | Mean Squares | F-Ratio | p-Value |  |
| Regression | $1.3063 \mathrm{E}+0081$ | 1.3063E+008 | 100.1291 | 0.0000 |  |
| Residual | 8.6103E+00766 | 1304593.7089 |  |  |  |


| Test for Normality |  |
| :--- | :--- |
|  | Test Statistic |
|  | p-Value |
| K-S Test (Lilliefors) | 0.0775 |

$\leftarrow$ KS test of normality assumption for residuals

The regression equation from the above analysis and represented on the graph is:
EGGNO = -4914.6 + 561.6 FEMLEN

In the regression equation, note that ' X ' and ' Y ' are replaced with the specific variables in question, i.e., FEMLEN and EGGNO. Also note that the dependent variable, EGGNO, is plotted on the Y axis, and the independent variable, FEMLEN, is plotted on the X axis. Another way of stating this is, "EGGNO is plotted against FEMLEN", or "EGGNO is regressed on FEMLEN."


## Example problems

1. The following data are rate of oxygen consumption ( $\mathrm{ml} / \mathrm{g} / \mathrm{hr}$ ) in crows at different temperatures $\left({ }^{\circ} \mathrm{C}\right)$. Does temperature affect oxygen consumption in crows? Determine the equation for predicting oxygen consumption from temperature. (Protocol link)

| temp | -18 | -15 | -10 | -5 | 0 | 5 | 10 | 19 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| oxygen | 5.2 | 4.7 | 4.5 | 3.6 | 3.4 | 3.1 | 2.7 | 1.8 |

2. Use the following data on mean adult body weight ( mg ) and larval density (no. $/ \mathrm{mm}^{3}$ ) of fruit flies to determine if there is a functional relationship between adult body mass and the density at which it was reared. Determine the equation for predicting body weight from larval density. (Protocol link)

| density | 1 | 3 | 5 | 6 | 10 | 20 | 40 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| weight | 1.356 | 1.356 | 1.284 | 1.252 | 0.989 | 0.664 | 0.475 |

Extrapolation: linear regressions are statistically valid only within limits of the data (independent variable, X ); beyond data - do not know if relationship is linear

(b)


Figure 5-24 (a) The famous American Museum of Natural History reconstruction of jaws of the giant fossil shark Carcharodon megalodon. Overzealous preparators in the museum's Department of Vertebrate Palcontology (shown framed by their handiwork in a 1909 photograph) used the largest teeth they could find to make the most spectacular jaw. In living sharks, teeth diminish in size as they approach the corners of the jaw but the preparators used teeth primarily from the anterior positions of the upper jaw, much exaggerating the final reconstruction. Estimates of the length of Carcharodon megalodon have ranged from 18.2 to 30.6 m ( 60 to 100 ft ). (American Museum of Natural History.) (b) A regression of tooth size on actual body length for the living Carcharodon carcharias indicates by extrapolation that Carcharodon megalodon was "only" $13 \mathrm{~m}(43 \mathrm{ft})$ in length! (Randall, J.E., Science, 181:169-170. 13 July 1973.)

A regression of tooth size on actual body length for the living Carcharodon carcharias indicates by extrapolation (assuming continued linearity) that C. megalodon was "only" $13 \mathrm{~m}(43 \mathrm{ft})$ in length!

Model building in regression (goal is to build a better model by increasing $\mathrm{r}^{2}$; results in more accurate prediction)

## Data transformation

1. SYSTAT e.g.: calibrate transmitters; DEMO
2. Linear vs. $\log 10$ data regressions - note increase in $\underline{r}^{2}$ and linearity with $\log$ transformation



## Predicting dependent variable Y from independent variable $X$

## A. Linear $(Y, X)$ equations: $\quad Y=\mathbf{a}+\mathbf{b X}$

Example 1: using the regression equation $\mathrm{Y}=14.5+2.56 \mathrm{X}$, predict Y when $\mathrm{X}=63$
$\underline{Y}=14.5+2.56(63)=175.78$

Example 2: inverse prediction (predict X from Y ); $\mathrm{Y}=14.5+2.56 \mathrm{X}$; by algebraic manipulation Y-14.5 $=2.56 \mathrm{X} ;(\mathbf{Y - 1 4 . 5}) / \mathbf{2 . 5 6}=\mathrm{X}$
predict X when $\mathrm{Y}=175.78$ :

$$
\underline{X}=(175.78-14.5) / 2.56=63
$$

B. Semilog $(\log Y, X)$ equations: $\log \mathbf{Y}=\log \mathbf{a}+\mathbf{b X}$ (must take the inverse $\log$ of $\log \mathrm{Y}$ to get final answer on linear scale)

Example: using the regression equation $\log \mathrm{Y}=1.42234+0.047560 \mathrm{X}$, predict Y when $\mathrm{X}=12.1$
$\underline{\log Y}=1.42234+0.047560(12.1)=1.99782$ (calculate regression coefficients and
answer to at least $\underline{\mathbf{5}}$ decimal places); inverse $\log 1.99782=\underline{\mathbf{9 9 . 4 9}}$
Note that the intercept (1.42234) is a $\log$ value (i.e., $\log a=1.42234$ ). You must not take the $\log$ of this value when calculating log $Y$; that would be the equivalent of taking the log of a log!
C. Log-log $(\log Y, \log X)$ and exponential equations: $\log Y=\log a+b(\log X) ; Y=a X^{b}$

Example 1 (logarithmic form): using the regression equation $\log \mathrm{Y}=2.53403+0.72000(\log \mathrm{X})$, predict Y when $\mathrm{X}=1.98$
$\underline{\log Y}=2.53403+0.72000(\log 1.98)=2.74763$ (calculate regression coefficients and answer to at least $\underline{5}$ decimal places)
inverse $\log 2.74763=\underline{\mathbf{5 5 9 . 2 8}}$

The most common form of the log-log regression equation, and one that is much easier to use is the exponential form:
$\log \mathrm{Y}=\log \mathrm{a}+\mathrm{b}(\log \mathrm{X})=\log \mathrm{a}+\log \mathrm{x}^{\mathrm{b}} ;$ take inverse $\log \mathrm{s}: \underline{\mathbf{Y}=\mathbf{a X}^{\mathbf{b}}}$ (exponential form)

- Example 2 (exponential form): using the regression equation $\mathrm{Y}=342 \mathrm{X}^{0.720}$, predict Y when
$X=1.98 ;$ *note that $342=$ the inverse $\log$ of 2.53403
$\underline{Y}=342\left(1.98^{0.720}\right)=559.28$


## Examples of important uses of exponential regressions in biology

1. Ecology: species-area curves (Isle Biogeography Theory)


> Common slope in some $(\sim 0.3)$
> -West Indian snakes: $S=1.19 \mathrm{~A}^{0.33}$
> -Galapagos land plants: $S=28.6 \mathrm{~A}^{0.32}$
> -Sierra Nevada mammals: $S=1.18 \mathrm{~A}^{0.32}$
2. Morphology: effects of scaling; e.g., brain size


Physiology: effects of scaling; e.g., metabolic rate and body mass




## Advanced statistical procedures commonly seen in the literature

1. Analysis of Covariance (ANCOVA)

- Method of comparing regression lines: eg,
-marsupials: $\mathrm{MR}=0.409 \mathrm{M}^{0.75}$
-eutherians MR $=0.676 \mathrm{M}^{0.75}$ (>60\% higher)
- Detect differences among means of two or more groups when the dependent variable is affected by a third (continuous) variable (=covariate)
- A covariate adds unwanted variability to the dependent variable. ANCOVA removes that variability and yields least squares means (means adjusted for the covariate effect)
- ANCOVA combines the use of both ANOVA and regression methods

Example1: A common belief is that men are stronger than women. Is this belief due to men being bigger or are men actually stronger when compared to women of similar body size? Test this question on data from a sample of healthy young adults (stronger.syz). The variables are sex, lean body mass, and a measure of strength called "slow, right extensor knee peak torque."

Add Example problems
2. Circular statistics (Raleigh Test) - techniques for data measured on an angular scale. Angular scales are circular in nature, have no designated zero, and the designation of high and low values is arbitrary. For example, $0^{\circ}$ and $360^{\circ}$ point to the same direction.
3. Principal component analysis (PCA) - variable reduction technique that describe variability among multiple observed variables in terms of a lower number of non-measured derived variables
4. MANOVA (multivariate analysis of variance) - a generalized form of ANOVA in which there are two or more independent and/or two or more dependent variables. MANOVA assesses main effects and possible interactions among the dependent variables and among the independent variables
5. Repeated measures ANOVA - each individual is measured $\geq$ two times
6. Logistic regression - regression with a binary dependent variable (e.g., presence/absence
7. Non-linear regression
8. Multiple regression - regression with $>1$ independent variable (Fig. 2)


## Statistical Tables

TABLE A. 1 AREAS OF THE NORMAL DISTRIBUTION

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0754 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2258 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2518 | 0.2549 |
| 0.7 | 0.2580 | 0.2612 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2996 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1 | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2 | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3 | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |
| 3.5 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 |
| 3.6 | 0.4998 | 0.4998 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.7 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.8 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.9 | 0.49995 | 0.49995 | 0.49996 | 0.49996 | 0.49996 | 0.49996 | 0.49996 | 0.49996 | 0.49997 | 0.49997 |

Table A. 3 Critical values of the chi-square distribution

| $\mathbf{d f}$ | $\mathbf{0 . 0 5}$ | $\mathbf{0 . 0 2}$ | $\mathbf{0 . 0 1}$ | $\mathbf{0 . 0 0 1}$ | $\mathbf{0 . 0 0 0 1}$ |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 3.84 | 5.41 | 6.63 | 10.83 | 15.14 |
| 2 | 5.99 | 7.82 | 9.21 | 13.82 | 18.42 |
| 3 | 7.81 | 9.84 | 11.34 | 16.27 | 21.11 |
| $\mathbf{4}$ | 9.49 | 11.67 | 13.28 | 18.47 | 23.51 |
| 5 | 11.07 | 13.39 | 15.09 | 20.51 | 25.74 |
| 6 | 12.59 | 15.03 | 16.81 | 22.46 | 27.86 |
| 7 | 14.07 | 16.62 | 18.48 | 24.32 | 29.88 |
| 8 | 15.51 | 18.17 | 20.09 | 26.12 | 31.83 |
| 9 | 16.92 | 19.68 | 21.67 | 27.88 | 33.72 |
| 10 | 18.31 | 21.16 | 23.21 | 29.59 | 35.56 |
| 11 | 19.68 | 22.62 | 24.72 | 31.26 | 37.37 |
| 12 | 21.03 | 24.05 | 26.22 | 32.91 | 39.13 |
| 13 | 22.36 | 25.47 | 27.69 | 34.53 | 40.87 |
| 14 | 23.68 | 26.87 | 29.14 | 36.12 | 42.58 |
| 15 | 25.00 | 28.26 | 30.58 | 37.70 | 44.26 |
| 16 | 26.30 | 29.63 | 32.00 | 39.25 | 45.92 |
| 17 | 27.59 | 31.00 | 33.41 | 40.79 | 47.57 |
| 18 | 28.87 | 32.35 | 34.81 | 42.31 | 49.19 |
| 19 | 30.14 | 33.69 | 36.19 | 43.82 | 50.80 |
| 20 | 31.41 | 35.02 | 37.57 | 45.31 | 52.39 |
| 21 | 32.67 | 36.34 | 38.93 | 46.80 | 53.96 |
| 22 | 33.92 | 37.66 | 40.29 | 48.27 | 55.52 |
| 23 | 35.17 | 38.97 | 41.64 | 49.73 | 57.08 |
| 24 | 36.42 | 40.27 | 42.98 | 51.18 | 58.61 |
| 25 | 37.65 | 41.57 | 44.31 | 52.62 | 60.14 |
| 26 | 38.89 | 42.86 | 45.64 | 54.05 | 61.66 |
| 27 | 40.11 | 44.14 | 46.96 | 55.48 | 63.16 |
| 28 | 41.34 | 45.42 | 48.28 | 56.89 | 64.66 |
| 29 | 42.56 | 46.69 | 49.59 | 58.30 | 66.15 |
| 30 | 43.77 | 47.96 | 50.89 | 59.70 | 67.63 |

Table Critical Values of Spearman's Rank Correlation Coefficient.

|  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $n$ | $\alpha=.05$ | $\alpha=.025$ | $\alpha=.01$ | $\alpha=.005$ | $n$ | $\alpha=.05$ | $\alpha=.025$ | $\alpha=.01$ | $\alpha=.005$ |
| 5 | . 900 |  |  |  | 18 | . 399 | 476 | . 564 | . 625 |
| 6 | . 829 | . 886 | . 943 |  | 19 | . 388 | . 462 | . 549 | . 608 |
| 7 | . 714 | . 786 | . 893 |  | 20 | . 377 | . 450 | . 534 | . 591 |
| 8 | . 643 | . 738 | . 833 | . 881 | 21 | . 368 | . 438 | . 521 | . 576 |
| 9 | . 600 | . 683 | . 783 | . 833 | 22 | . 359 | . 428 | . 508 | . 562 |
| 10 | . 564 | . 648 | . 745 | . 794 | 23 | . 351 | . 418 | . 496 | . 549 |
| 11 | . 523 | . 623 | . 736 | . 818 | 24 | . 343 | . 409 | . 485 | . 537 |
| 12 | 497 | . 591 | . 703 | . 780 | 25 | . 336 | . 400 | . 475 | . 526 |
| 13 | 475 | . 566 | . 673 | . 745 | 26 | . 329 | . 392 | . 465 | . 515 |
| 14 | 457 | . 545 | . 646 | . 716 | 27 | . 323 | . 385 | . 456 | . 505 |
| 15 | 441 | . 525 | . 623 | . 689 | 28 | . 317 | . 377 | . 448 | . 496 |
| 16 | 425 | . 507 | . 601 | . 666 | 29 | . 311 | . 370 | . 440 | . 487 |
| 17 | 412 | . 490 | . 582 | . 645 | 30 | . 305 | . 364 | . 432 | 478 |

Table A. 2 Critical values of the $t$ distribution

| df | $\alpha$ (Two-Tailed) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.2 | 0.1 | 0.05 | 0.02 | 0.01 | 0.001 | 0.0001 |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 636.619 | 6366.198 |
| 2 | 1.886 | 2.920 | 4.303 | 6.695 | 9.925 | 31.598 | 99.992 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 12.924 | 28.000 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 8.610 | 15.544 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 6.869 | 11.178 |
| 6 | 1.44 | 1.943 | 2.447 | 3.143 | 3.707 | 5.959 | 9.082 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 5.408 | 7.885 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 5.041 | 7.120 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.781 | 6.594 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.587 | 6.211 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.437 | 5.921 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 4.318 | 5.694 |
| 13 | 1.35 | 1.771 | 2.160 | 2.650 | 3.012 | 4.221 | 5.513 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 4.140 | 5.363 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 4.073 | 5.239 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 4.015 | 5.134 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.965 | 5.044 |
| 18 | 1.33 | 1.734 | 2.101 | 2.552 | 2.878 | 3.922 | 4.966 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.883 | 4.897 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.850 | 4.837 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.819 | 4.784 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.792 | 4.736 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.767 | 4.693 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.745 | 4.654 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.725 | 4.619 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.707 | 4.587 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.690 | 4.558 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.674 | 4.530 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.659 | 4.506 |
| 30 | 1.31 | 1.697 | 2.042 | 2.457 | 2.750 | 3.646 | 4.482 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.551 | 4.321 |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.460 | 4.169 |
| 100 | 1.292 | 1.660 | 1.984 | 2.364 | 2.626 | 3.390 | 4.053 |
| $\infty$ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.291 | 3.750 |

Table A. 6 Critical values of the $F$ distribution $\left(\alpha=0.05 ; \mathrm{df}_{1}=\right.$ treatment degrees of freedom, $\mathrm{df}_{2}=$ error degrees of freedom). Part A: $\alpha=0.05$

| df $_{2}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 18.5 | 19.0 | 19.2 | 19.3 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | $\mathbf{1 9 . 4}$ | $\mathbf{1 9} 9$ |  |
| 3 | 10.1 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.76 | 8.74 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.93 | 5.91 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.71 | 4.68 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.03 | 4.00 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.77 | 3.73 | 3.68 | 3.64 | 3.60 | 3.57 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.31 | 3.28 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.10 | 3.07 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.94 | 2.91 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.82 | 2.79 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.72 | 2.69 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.51 | 2.48 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.31 | 2.28 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.21 | 2.16 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.13 | 2.09 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.04 | 2.04 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.95 | 1.92 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.17 | 2.09 | 2.02 | 1.96 | 1.91 | 1.87 | 1.83 |

